This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

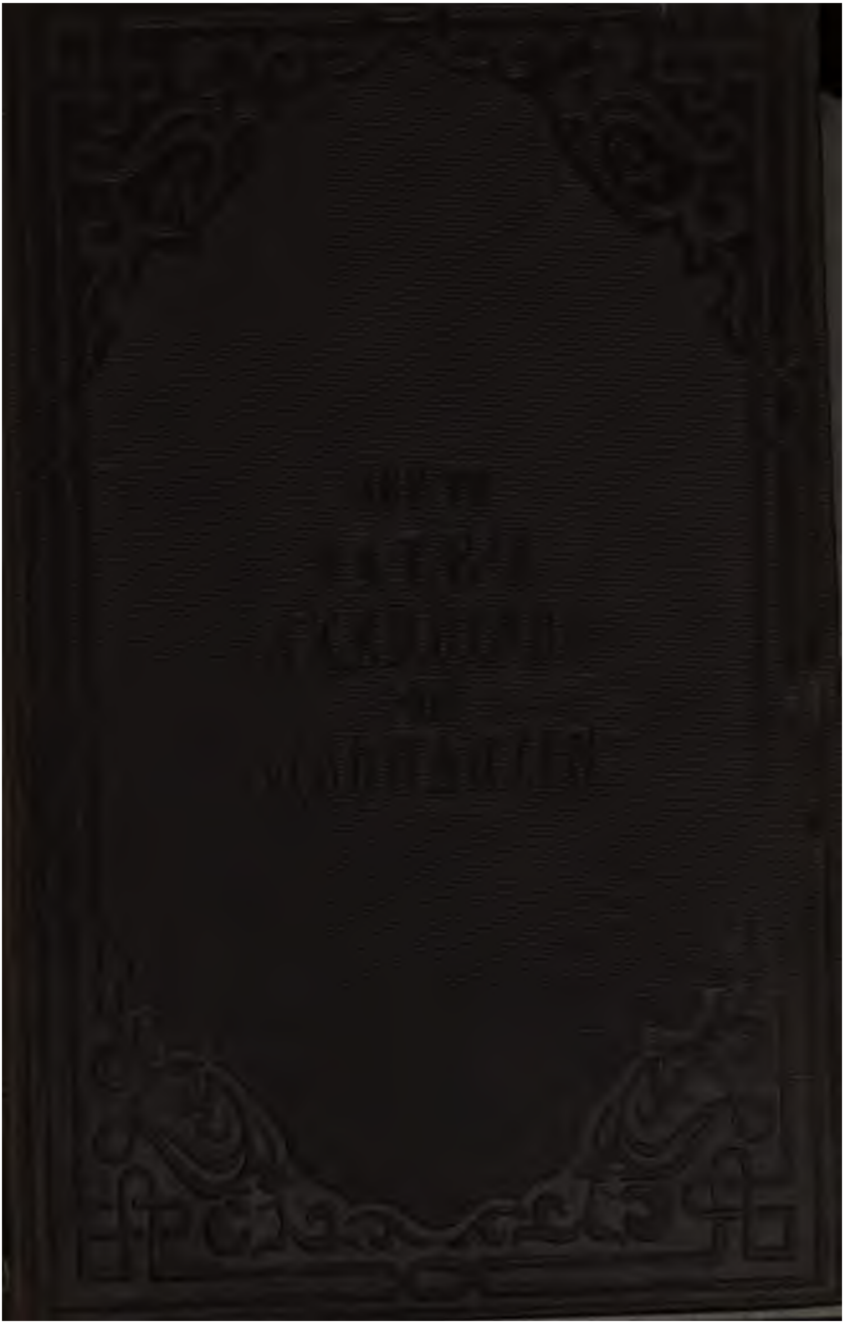
+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`



ESTABLISHED SCHOOL BOOKS.

I.

The Rev. J. E. RIDDLE's COPIOUS and CRITICAL LATIN-ENGLISH LEXICON. *Second Edition.* Post 4to. 50s.

II.

The Rev. J. E. RIDDLE's COMPLETE LATIN-ENGLISH and ENGLISH-LATIN DICTIONARY. New Editions. 8vo. 31s. 6d.

Separately { The Latin-English Dictionary, 21s.
The English-Latin Dictionary, 10s. 6d.

III.

The Rev. J. E. RIDDLE's YOUNG SCHOLAR'S LATIN-ENGLISH and ENGLISH-LATIN DICTIONARY. New Editions. Square 12mo. 12s.

Separately { The Latin-English Dictionary, 7s.
The English-Latin Dictionary, 5s. 6d.

IV.

The Rev. J. E. RIDDLE and the Rev. T. K. ARNOLD's COPIOUS and CRITICAL ENGLISH-LATIN LEXICON. New Edition. 8vo. 25s.

V.

Mr. A. RICH's ILLUSTRATED COMPANION to the LATIN DICTIONARY and GREEK LEXICON. With 2,000 Classical Woodcuts. Post 8vo. 21s.

VI.

Mr. C. D. YONGE's NEW LATIN GRADUS AD PARNASSUM, containing every Word used by the Poets of good Authority. New Edition. Post 8vo. 9s.

VII.

Mr. C. D. YONGE's EXERCISES in LATIN PROSE COMPOSITION. 12mo. 5s. 6d.

VIII.

Mr. C. D. YONGE's NEW ENGLISH-GREEK LEXICON, containing all the Greek Words used by Writers of good Authority. Post 4to. 21s.

IX.

Mr. C. D. YONGE's EXERCISES in GREEK PROSE COMPOSITION. 12mo. 5s. 6d.

X.

ZUMPT's LATIN GRAMMAR. Translated with the Author's co-operation and final corrections, by Dr. L. Schmitz. Second Edition. 8vo. 14s.

London: Longman, Brown, Green, and Longmans.

KEY

TO

TATE'S EXERCISES ON MECHANICS

AND

NATURAL PHILOSOPHY.

BIBLIOTHECA·BODLEIANA

BY

THOMAS TATE, F.R.A.S.

AUTHOR OF
"A TREATISE ON THE FIRST PRINCIPLES OF ARITHMETIC,"
"ALGEBRA MADE EASY," "THE ELEMENTS OF MECHANISM,"
"THE PRINCIPLES OF GEOMETRY AND MENSURATION,"
ETC. ETC.

LONDON:
LONGMAN, BROWN, GREEN, AND LONGMANS.
1852.

186. c. 30.

London:

Spottiswoodes and Shaw,
New-street-Square.

CONTENTS.

Page
Unit of work 1
Work of living agents 6
Work in overcoming the resistance of friction 8
Work of the saw machine 10
Work in moving a body on an inclined plane 11
Additional exercises 14
Work on raising material having a given form 16
Work in excavations 21
Work in overcoming the resistance of fluids 23
Modulus of a machine 25
The water wheel. Hydraulic ram 27—29
Additional exercises 30
Work of steam having a mean pressure 31
Work of steam in relation to the water evaporated 32
Work of steam used expansively 37
Additional exercises 41
Maximum work of steam 42
Transmission of work by simple machines 44
The lever 44
To graduate the lever of a safety valve, &c. 47
Centre of gravity 51
Wheel and axle. Cogged wheels 52
Compound wheel and axle 54
The pulley 55
Inclined plane and wedge 56—57
The screw—compound screw—endless screw 58—59
Hydrostatic press 60
Accumulated work. Gravity 61
Additional exercises 65
Work in a rotating body. Centre of gyration 66

	Page
Maximum velocity of a piston	68
Velocity acquired in descending an inclined plane	69
The fire engine. Poncelet's water wheel	70
The fly wheel	71
Equilibrium of pressures	72
Modulus of stability. Pressure of roofs	73—74
Embankments and revetment walls	75
Floating bodies. Specific gravity	82
Limiting angle of resistance	84
Equilibrium of the lever, &c., taking friction into account	84
Force of traction. Least traction	87
Eqqilibrium of the arch	88
Strength of material	92
Examples for pupils	104

KEY

TO

TATE'S EXERCISES ON MECHANICS

AND

NATURAL PHILOSOPHY.

UNIT OF WORK.

Art. 2.

Ex. 3.

Weight of the ram in lbs. $= 14 \times 112 = 1568$.

Units of work in raising 1 lb. 19 ft. $= 19$.

" " " 1568 lbs. 19 ft. $= 1568$ times 19 $= 29792$.

Ex. 5.

Weight of coals in lbs. $= 3 \times 112 = 336$;

Depth of the pit in feet $= 25 \times 6 = 150$;

$\therefore$ Work $= 336 \times 150 = 50400$.

Ex. 8.

Weight of the water in lbs. $= 62{\cdot}5 \times 250 = 15625$;

Depth of the mine in ft. $= 36 \times 6 = 216$ ft.;

$\therefore$ Work $= 15625 \times 216 = 3375000$.

Ex. 10.

Space moved over per min. $=\frac{3\times5280}{60}=264$ ft.

$\therefore$ Work per min. $=130\times264=34320$.

Art. 3.

Ex. 2.

Weight of coals in lbs. $=5\times112=560$ lbs.

Depth of the pit in ft. $=160\times6=960$ ft.

As 560 lbs. of coals are raised every minute,

$\therefore$ Work done per min. $=560\times960=537600$.

$\therefore$ Horse powers or H P $=\frac{537600}{33000}=16{\cdot}2^{+}$.

Ex. 4.

Weight of water in lbs. $=62{\cdot}5\times2500=156250$ lbs.

Depth of the mine in ft. $=86\times6=516$ ft.

As 156250 lbs. of water are raised every hour,

$\therefore$ Work per hour $=156250\times516=80625000$;

$\therefore$ Work per min. $=\frac{80625000}{60}=1343750$;

$\therefore$ H. P. $=\frac{1343750}{33000}=40{\cdot}7^{+}$

Ex. 6.

Work done by the horses per hour $=33000\times3\times60=5940000$;

Depth of the pit in ft. $=40\times6=240$ ft.

Work in raising 1 lb. of coals $=1\times240=240$.

Now, as 240 units of work go to raise 1 lb. of coals, therefore as many times as 240 can be taken out of the whole work, that is, 5940000, so many lbs. must be raised in the time.

$\therefore$ No. lbs. raised per hour $= \frac{5940000}{240} = 24750$ lbs. or 11·04 tons.

Or thus,

Let $x =$ no. tons raised per hour; then,
Wt. coals in lbs. $= 2240 \times x$.
$\therefore$ Work in raising coals per hour $= 2240 \times x \times 240$.

Now the horses have to perform this work; but from the foregoing solution, the work of the horses per hour is 5940000.

$\therefore 2240 \times x \times 240 = 5940000.$

$\therefore x = \frac{5940000}{2240 \times 240} = 11{\cdot}04$ tons.

Ex. 8.

Work of the engine per min. $= 33000 \times 7 = 231000$.
Work in raising 1 ton $= 2240 \times 47 \times 6 = 631680$.

$\therefore$ No. minutes to raise 1 ton $= \frac{631680}{231000} = 2{\cdot}7^{+}$

Or thus,

Let $x =$ the no. minutes to raise 1 ton.
By the foregoing solution, we have,
Work engine per min. $= 231000$.
$\therefore$ „ in x min. $= 231000 \times x$

Now the engine has to raise 1 ton of coals in x minutes; hence the work of the engine in this time must be equal to the work in raising 1 ton of coals.

$\therefore 231000 \times x = 631680.$

$\therefore x = \frac{631680}{231000} = 2{\cdot}7$ min.,

which is the time required in the question.

Ex. 10.

Work done per hour $= 33000 \times 34 \times 60$.
Work in raising 1 foot of water $= 62{\cdot}5 \times 72 \times 6$.

$\therefore$ No. ft. raised per hour $= \dfrac{33000 \times 34 \times 60}{62{\cdot}5 \times 72 \times 6} = 2493.$

Or thus,

Let $x =$ No. c. ft. water raised per hour; then,

Work requisite in raising the water per hour $= 62{\cdot}5 \times 72 \times 6 \times x$.

Work engine per hour $= 33000 \times 34 \times 60$.

$\therefore 62{\cdot}5 \times 72 \times 6 \times x = 33000 \times 34 \times 60.$

$\therefore x = \dfrac{33000 \times 34 \times 60}{62{\cdot}5 \times 72 \times 6} = 2493$ feet.$^{+}$

Ex. 13.

Work done per min. $= \dfrac{2240 \times 5 \times 210 \times 6}{60} = 235200$

Useful work of one H. P. per min. $= \dfrac{4}{5}$ of $33000 = 26400.$

And H. P. $= \dfrac{235200}{26400} = 8{\cdot}9.^{+}$

Ex. 15.

No. gals. water pumped per min. $= \dfrac{5000 \times 40}{8 \times 60} = 416\frac{2}{3}$ gals.

Weight water pumped per min. $= 416\frac{2}{3} \times 10 = 4166\frac{2}{3}$ lbs.

Work per min. $= 4166\frac{2}{3} \times 60 = 250000.$

$\therefore$ H. P. $= \dfrac{250000}{33000} = 7{\cdot}5^{+}.$

Ex. 17.

Work per min. in the 1st level $= 62{\cdot}5 \times 30 \times 20 \times 6.$

„ „ 2nd „ $= 62{\cdot}5 \times 20 \times 50 \times 6.$

„ „ 3rd „ $= 62{\cdot}5 \times 10 \times 70 \times 6.$

$\therefore$ Total work per min. $= (30 \times 20 + 20 \times 50 + 10 \times 70)\, 62{\cdot}5 \times 6 = 2300 \times 62{\cdot}5 \times 6 = 862500.$

$\therefore$ H. P. $= \dfrac{862500}{33000} = 26{\cdot}1^{+}.$

Ex. 18.

Here as $\frac{1}{3}$ of the work of the engine is lost in transmission, the effective work must be equal to $\frac{2}{3}$ of the work of the engine, that is,

$\frac{2}{3}$ of H. P. engine $= 26{\cdot}1^{+}$.

$$\therefore \text{H. P. engine} = \frac{26{\cdot}1^{+} \times 3}{2} = 39{\cdot}2 \text{ nearly.}$$

Ex. 19.

Work engine per hour $= 33000 \times 12 \times 60$.

But since only $\frac{2}{3}$ of this work goes to pump the water, we have

Effective work engine in pumping the water $= \frac{2}{3}$ of $33000 \times 12 \times 60 = 15840000$.

Work in pumping 1 c. ft. of water $= 62{\cdot}5 \times 40 \times 6 = 15000$.

$$\therefore \text{No. c. ft. water} = \frac{\text{effective work engine}}{\text{work in pumping 1 c. ft.}} = \frac{15840000}{15000}$$

$= 1056$ c. ft.

Or thus,

Let $x =$ no. cubic feet of water raised per hour; then wt. water in lbs. $= 62{\cdot}5 \times x$;

$\therefore$ Work in pumping water per hour $= 62{\cdot}5 \times x \times 40 \times 6$.

Now from the foregoing solution the effective work of the engine per hour is 15840000.

$\therefore$ $62{\cdot}5 \times x \times 40 \times 6 = 15840000$.

$$\therefore x = \frac{15840000}{62{\cdot}5 \times 40 \times 6} = \frac{15840000}{15000} = 1056 \text{ cubic feet.}$$

Ex. 22.

Weight of hammer in lbs. $= 10 \times 112 = 1120$ lbs.

Work in each lift $= 1120 \times 3 = 3360$.

Work in 70 lifts, or per min. $= 3360 \times 70 = 235200$.

$$\therefore \text{H. P.} = \frac{235200}{33000} = 7{\cdot}1^{+}.$$

Ex. 23.

Work done per min. $=33000 \times 4 = 132000$.

Work done per min. in raising 1 lb. of the hammer $=1 \times 3 \times 70 = 210$.

$\therefore$ No. lbs. weight in the hammer $= \dfrac{\text{work done per min.}}{\text{work done per min. on 1 lb.}}$

$= \dfrac{132000}{210} = 628{\cdot}6$ lbs. or $5{\cdot}6$ cwts.

Or thus,

Let x = no. lbs. weight in the hammer; then

Work in 1 lift $= x \times 3$;

Work in 70 lifts, or work per min. $= x \times 3 \times 70$.

Now this work has to be done by an engine of 4 horse powers;

$\therefore\ x \times 3 \times 70 = 33000 \times 4$;

$\therefore\ x = \dfrac{33000 \times 4}{3 \times 70} = 628{\cdot}6$ lbs. or $5{\cdot}6$ cwts.

WORK OF LIVING AGENTS.

Art. 5.

Ex. 7.

Work of the man per min. $= 2600$.

" " in a day of 8 hours long $= 2600 \times 60 \times 8 = 1248000$.

Work in pumping 1 c. ft. of water $= 62{\cdot}5 \times 50 = 3125$.

$\therefore$ No. c. ft. water pumped per day $= \dfrac{\text{work per day}}{\text{work in 1 c. ft. water}}$

$= \dfrac{1248000}{3125} = 399{\cdot}3$.

Or thus,

Let x = no. c. ft. water pumped per day; then, Wt. water in lbs. $= 62{\cdot}5 \times x$.

Work in pumping water per day $= 62{\cdot}5 \times x \times 50 \times 8$.

But by the foregoing solution the work of the man per day is 1248000;

$$\therefore\ 62{\cdot}5 \times x \times 50 \times 8 = 1248000,$$

$$\therefore\ x = \frac{1248000}{62{\cdot}5 \times 50 \times 8} = 399{\cdot}3 \text{ c. ft.}$$

Ex. 9.

Here the number of tons which the man raises in a day is 3·7,

$$\therefore \text{ Cost per ton} = \frac{\text{wages per day}}{\text{no. tons raised per day}} = \frac{30 \text{ d.}}{3{\cdot}7} = 8d., \text{ and}$$

about half a farthing.

Ex. 10.

Work of the horse per day $= 22000 \times 60 \times 8 = 10560000$.

Work in raising 1 ton of coals $= 2240 \times 25 \times 6 = 336000$.

No. tons raised by the horse per day $= \dfrac{\text{work per day}}{\text{work in 1 ton}} = \dfrac{10560000}{336000} = 31{\cdot}42$,

$$\therefore \text{ Cost per ton} = \frac{8s.}{31{\cdot}42} = 3d. \text{ nearly.}$$

Ex. 11.

Here an ass is assumed to do $\frac{1}{5}$ of the work of a horse, and therefore two asses will do $\frac{2}{5}$ of the work of a horse; but from the last exercise the number of tons raised by a horse in a day is 31·42.

$$\therefore \text{ No. tons raised by the asses per day} = \frac{2}{5} \text{ of } 31{\cdot}42 = 12{\cdot}56.$$

$$\therefore \text{ Cost per ton} = \frac{3s.}{12{\cdot}56} = 2\tfrac{3}{4}d.^{+}$$

WORK IN OVERCOMING THE RESISTANCE OF FRICTION.

Art. 8.

Work in moving a carriage upon a horizontal plane.

Ex. 2.

Resistance of friction $= 8 \times 60 = 480$ lbs.

Distance moved over in ft. per min. $= \dfrac{50 \times 5280}{60} = 4400$ ft.

$\therefore$ Work of friction per min. $= 480 \times 4400 = 2112000$.

$\therefore$ H.P. $= \dfrac{2112000}{33000} = 64$.

Ex. 4.

Work done by the engine per hour $= 60 \times 33000 \times 60$.

Work consumed in moving the train over 1 mile $= 8 \times 90 \times 5280$.

$\therefore$ No. miles per hour $= \dfrac{60 \times 33000 \times 60}{8 \times 90 \times 5280}$ (by cancelling)

$$= \frac{\overset{\cancel{2}}{\cancel{6}}\cancel{0} \times \overset{11000}{\cancel{3}\cancel{3}\cancel{0}\cancel{0}\cancel{0}} \times \cancel{6}\cancel{0}}{\underset{4}{\cancel{8}} \times \underset{\cancel{3}\cancel{0}}{\cancel{9}\cancel{0}} \times \underset{88}{\cancel{5}\cancel{2}\cancel{8}\cancel{0}}} = \frac{11000}{352} = 31{\cdot}2.$$

Or thus,

Let $x =$ the rate in miles per hour ; then,

Distance moved over in feet per hour $= x \times 5280$;

Resistance friction in lbs. $= 8 \times 90$;

$\therefore$ Work due to friction per hour $= 8 \times 90 \times x \times 5280$.

But this must be equal to the work done by the engine per hour ;

$$\therefore 8 \times 90 \times x \times 5280 = 60 \times 33000 \times 60,$$

$$\therefore x = \frac{60 \times 33000 \times 60}{8 \times 90 \times 5280} = 31{\cdot}2^{+} \text{ miles.}$$

Ex. 6.

Work engine per hour $= 80 \times 33000 \times 60$.

Work consumed per hour in moving 1 ton $= 8 \times 45 \times 5280$.

$$\therefore \text{ No. tons } = \frac{\text{Work engine per hour}}{\text{Work in 1 ton per hour}} = \frac{80 \times 33000 \times 60}{8 \times 45 \times 5280}$$

$$= \frac{10 \times 11000^{*}}{15 \times 88} = \frac{110000}{1320} = 83{\cdot}3^{+}.$$

Or thus,

Let $x =$ the gross weight of the train in tons; then,
Resistance due to friction in lbs. $= 8 \times x$;
Distance moved over in feet per hour $= 45 \times 5280$;
$\therefore$ Work of friction per hour $= 8 \times x \times 45 \times 5280$.
But the work of the engine per hour $= 80 \times 33000 \times {\cdot}60$.
$\therefore\ 8 \times x \times 45 \times 5280 = 80 \times 33000 \times 60$;

$$\therefore\ x = \frac{80 \times 33000 \times 60}{8 \times 45 \times 5280} = 83{\cdot}3^{+} \text{ tons.}$$

Ex. 8.

Work expended in moving the train over 330 miles $= 8 \times 70 \times 330 \times 5280$.

Work engine per hour $= 60 \times 33000 \times 60$.

$\therefore$ No. of hours in moving over 330 miles $=$

$$\frac{\text{work in distance}}{\text{work engine per hour}} = \frac{8 \times 70 \times 330 \times 5280}{60 \times 33000 \times 60} = \frac{8 \times 70 \times 88^{*}}{60 \times 100}$$

$= 8{\cdot}2^{+}$.

Ex. 14.

Weight moved in lbs. $= 1\frac{3}{4} \times 2240 = 3920$ lbs.

Now the resistance of friction, in this case, is $\frac{1}{30}$ of this weight, and moreover the traction must be equal to the resistance;

$$\therefore \text{ Traction } = \frac{1}{30} \text{ of } 3920 \text{ lbs.} = 130\frac{2}{3} \text{ lbs.}$$

---

* By cancelling.

Ex. 17.

Weight of gross load $= 160 \times 30 = 4800$ lbs.

$\therefore$ Weight of net load $= \frac{4}{5}$ of 4800 lbs. $=$ 3840 lbs.

Weight of the material $= 24000 \times 120 = 2880000$ lbs.

$\therefore$ No. loads $= \frac{\text{wt. material}}{\text{wt. net load}} = \frac{2880000}{3840} = 750.$

No. hours in going and returning with each load $= \frac{3}{2\frac{1}{2}} + \frac{3}{5} = \frac{6}{5} + \frac{3}{5} = \frac{9}{5} = 1{\cdot}8$ hours.

$\therefore$ No. hours in removing the whole material $=$ no. hours with each load $\times$ No. loads $= 1{\cdot}8 \times 750 = 1350$ hours,

$\therefore$ No. of days' work $= \frac{1350}{8} = 168{\cdot}75$ days.

Cost $= 168{\cdot}75 \times 10s. = 84l.\ 7s.\ 6d.$

Work of the Saw Machine.

ART. 9.

Ex. 2.

Work in cutting 1 foot $= 29000$
Work in cutting 5600 feet $= 29000 \times 5600 = 162400000.$

$$\therefore \text{Work per min.} = \frac{162400000}{12 \times 60} = 225555.$$

And H. P. $= \frac{225555}{33000} = 6{\cdot}8^{+}.$

Ex. 4.

Work engine per min. $= 33000 \times 5 = 165000.$
Surface of the planks $= 20 \times 1\frac{1}{2} \times 300 = 9000$ sq. ft.
Work in cutting the planks $= 29000 \times 9000 = 261000000.$

$\therefore$ No. minutes in cutting $= \frac{261000000}{165000} = 1581{\cdot}8$ min., or 26·3 hours.$^{+}$

WORK IN MOVING A BODY ON AN INCLINED PLANE.

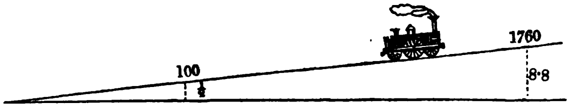
Work in overcoming the resistances of friction and gravity.

ART. 11.

Ex. 2.

Speed train per min. $= \dfrac{20 \times 5280}{60} = 20 \times 88 = 1760$ feet.

Weight train in lbs. $= 90 \times 2240 = 201600$ lbs.



Rise of the rail in 100 feet $= \frac{1}{2}$ ft.

„ „ 1 foot $= \frac{1}{200}$ ft.

„ „ 1760 feet $= \frac{1760}{200}$ ft. $= 8{\cdot}8$ ft.

Here the whole weight of the train is raised 8·8 ft. every minute in opposition to gravity;

∴ Work due to gravity per min. = weight train in lbs. × vertical height raised $= 201600 \times 8{\cdot}8 = 1774080$.

Work due to friction per min. $= 8 \times 90 \times 1760 = 1267200$,

∴ Total work per min. = work due to gravity + work due to friction $= 1774080 + 1267200 = 3041280$.

And H. P. $= \dfrac{3041280}{33000} = 92$.

Ex. 4.

Rise rail in 1 foot $= \dfrac{1}{400}$ ft.

Rise rail in 5280 feet, or 1 mile $= \dfrac{1 \times 5280}{400} = 13{\cdot}2$ feet.

Weight train in lbs. $= 60 \times 2240 = 134400$ lbs.

$\therefore$ Work due to gravity in 1 mile $= 134400 \times 13{\cdot}2 = 1774080$.

$\therefore$ Work due to friction in 1 mile $= 8 \times 60 \times 5280 = 2534400$.

$\therefore$ Total work done in 1 mile $= 1774080 + 2534400 = 4308480$.

Work engine per hour $= 40 \times 33000 \times 60 = 79200000$.

$\therefore$ No. miles per hour $= \dfrac{\text{Work engine}}{\text{Work in 1 mile}} = \dfrac{79200000}{4308480} =$ 18·4 nearly.

Or thus,

Let $x =$ no. miles moved over per hour; then proceeding as in the foregoing solution, we have

Rise rail in 1 mile $= 13{\cdot}2$ feet,

$\therefore$ „ x miles $= 13{\cdot}2 \times x$ feet.

Wt. train in lbs. $= 134400$ lbs.

Work due to gravity per hour $= 134400 \times 13{\cdot}2 \times x$.

Resistance friction in lbs. $= 8 \times 60$;

Distance moved over in feet per hour $= x \times 5280$.

$\therefore$ Work due to friction per hour $= 8 \times 60 \times x \times 5280$.

Total work due to the resistances per hour = work due to gravity + work due to friction $= 134400 \times 13{\cdot}2 \times x + 8 \times 60 \times x \times 5280$.

But this must be equal to the work done by the engine per hour:

$$\therefore 134400 \times 13{\cdot}2 \times x + 8 \times 60 \times x \times 5280 = 79200000,$$

$$\therefore x\,(134400 \times 13{\cdot}2 + 8 \times 60 \times 5280) = 79200000,$$

$$\therefore x = \frac{79200000}{134400 \times 13{\cdot}2 + 8 \times 60 \times 5280} = \frac{79200000}{4308480} = 18{\cdot}4$$

miles.

Ex. 5.

Work engine per min. $= 50 \times 33000 = 1650000$.

Speed train per min. $= \dfrac{20 \times 5280}{60} = 20 \times 88 = 1760$ ft.

Rise rail in 1760 ft. $= \dfrac{3 \times 1760}{400} = 13{\cdot}2$ ft.

Work due to gravity per min. for 1 ton $= 2240 \times 13{\cdot}2 = 29568$.

Work due to friction per min. for 1 ton $= 8 \times 1760 = 14080$.

$\therefore$ Total work due to gravity and friction per min. for 1 ton $= 29568 + 14080 = 43648$.

$\therefore$ No. tons $= \dfrac{\text{work engine per min.}}{\text{work on 1 ton per min.}} = \dfrac{1650000}{43648} =$ 37·8 tons.

Or thus.

Let $x =$ no. tons weight in the train; then

Work due to friction per min. $= 8 \times x \times 1760$;

" " gravity " $= x \times 2240 \times 13{\cdot}2$;

$\therefore$ Total work due to the resistances per min. $= 8 \times x \times 1760 + x \times 2240 \times 13{\cdot}2$.

But the work of the engine per min. is 165000;

$$\therefore 8 \times x \times 1760 + x \times 2240 \times 13{\cdot}2 = 165000;$$

$$\therefore x = \frac{165000}{8 \times 1760 + 2240 \times 13{\cdot}2} = \frac{165000}{14080 + 29568} = 37{\cdot}8 \text{ tons.}$$

Ex. 7.

Speed train per min. $= \dfrac{50 \times 5280}{60} = 4400$.

Work due to friction per min. $= 8 \times 80 \times 4400 = 2816000$.

Work due to gravity per min. $= 80 \times 2240 \times 14\frac{2}{3} = 2628266$.

In this case gravity acts with the engine.

$\therefore$ Work engine per min. $=$ work friction $-$ work gravity $= 2816000 - 2628266 = 187734$.

And H. P. $= \dfrac{187734}{33000} = 5{\cdot}6+$.

Ex. 9.

Resistance of friction $=\frac{1}{30}$ of 2240 lbs. $=\frac{2240}{30}$ lbs.

Work due to friction in 100 ft. $=\frac{2240}{30}\times 100=7466{\cdot}6+$.

„ „ gravity „ $=2240\times 5=11200$.

Now as the work due to gravity is greater than the work due to friction, the horse must do work in a backward direction in taking the load down the hill;

$\therefore$ Work of the horse in 100 ft. $=11200-7466{\cdot}6=3733{\cdot}4$.

$\therefore$ Work of the horse in 1 ft. $=\frac{3733{\cdot}4}{100}=37{\cdot}334$ lbs.

But this work of the horse over 1 foot is obviously his backward pressure, because when a pressure of a certain number of lbs. is exerted over 1 foot, the units of pressure are equal to the units of work.

Or thus.

Let $x=$ no. lbs. backward pressure of the horse; then we have

Work of the horse in 100 ft. $=x\times 100$.

But the work of the horse over this space must be 3733·4 (see the foregoing solution).

$$\therefore\ x\times 100=3733{\cdot}4,$$

$$\therefore\ x=\frac{3733{\cdot}4}{100}=37{\cdot}334 \text{ lbs.}$$

Additional Exercises.

Ex. 12.

What must be the traction of a horse to draw a load of 2 tons up a hill having a rise of $\frac{1}{2}$ in 100, the co-efficient of friction being $\frac{1}{20}$?

Suppose the load to be moved over 100 ft.

Resistance friction in lbs. $= \frac{1}{20}$ of $2 \times 2240 = 224$ lbs.

Work due to friction $= 224 \times 100 = 22400$.

„ „ gravity $= 2 \times 2240 \times \frac{1}{2} = 2240$.

$\therefore$ Total work to be done by the horse $= 22400 + 2240 = 24640$.

But we have also,

$$\text{Work of horse} = \text{traction} \times 100.$$

$$\therefore \text{Traction} \times 100 = 24640;$$

$$\therefore \text{Traction} = \frac{24640}{100} = 246\cdot4 \text{ lbs.}$$

Ex. 13.

A horse can just draw a load of 1 ton up a hill (having a small inclination), while he can draw a load of $1\frac{1}{2}$ tons down the hill; it is required to find the rise of the hill for every 100 feet, when the co-efficient of friction is $\frac{1}{20}$.

Suppose the hill to be 100 ft. long, and let $x =$ the no. ft. rise of the hill in 100 ft.

Resistance friction up the hill $= \frac{1}{20}$ of 2240 lbs. $= 112$ lbs.

„ friction down the hill $= \frac{1}{20}$ of $1\frac{1}{2} \times 2240$ lbs. $= 168$ lbs.

Work due to gravity up the hill $= 2240 \times x$.

„ „ friction „ „ $= 112 \times 100$.

$\therefore$ Total work to be done by the horse up the hill $= 2240 \times x + 112 \times 100$.

Work due to gravity down the hill $= 1\frac{1}{2} \times 2240 \times x = 3360 \times x$.

Work due to friction down the hill $= 168 \times 100$.

$\therefore$ Total work to be done by the horse down the hill $= 168 \times 100 - 3360 \times x$.

Now since the horse is supposed to exert the same traction in going up the hill as he does in going down the hill, therefore he must do the same work in both cases; hence we have

$$2240 \times x + 112 \times 100 = 168 \times 100 - 3360 \times x,$$
$$\therefore\ 5600x = 5600,$$
$$\therefore\ x = 1 \text{ foot},$$

which is the rise of the hill for every 100 feet.

Ex. 14.

The work in moving a load (say of 1 ton) up a hill, of small inclination, is $1\frac{1}{2}$ times the work in moving the same load down the hill; required the rise of the hill, supposing the co-efficient of friction to be $\frac{1}{10}$.

Suppose the hill to be 100 feet long, and $x =$ the rise in this distance; then we have

Work up the hill $= \frac{1}{10}$ of $2240 \times 100 + 2240 \times x = 22400 + 2240 \times x$.

Work down the hill $= \frac{1}{10}$ of $2240 \times 100 - 2240 \times x = 22400 - 2240 \times x$.

Hence we have, by the question,

$$22400 + 2240 \times x = 1\tfrac{1}{2}\,(22400 - 2240 \times x),$$
$$\therefore\ 5600 \times x = 11200,$$
$$\therefore\ x = \frac{11200}{5600} = 2 \text{ feet}.$$

WORK IN RAISING MATERIAL HAVING A GIVEN FORM.

Art. 12.

Ex. 3.

By the table, Art. 5. p. 9., a man turning a handle can perform 2600 units of work per minute, working for 8 hours per day. Now the work to be done is 20357568; hence we have

Work of the man per day $= 2600 \times 60 \times 8 = 1248000$.

$$\therefore\ \text{No. days} = \frac{\text{whole work}}{\text{work done per day}} = \frac{20357568}{1248000} = 16{\cdot}3.$$

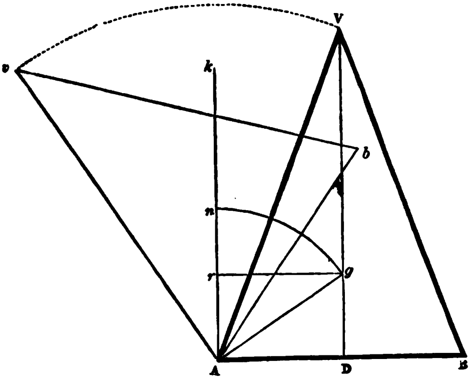
Or thus.

Let $x =$ the no. days for the man to complete the work; then we have

$$\text{Work of the man per day} = 1248000\,;$$
$$\therefore\ \text{Work of the man in } x \text{ days} = 1248000 \times x\,;$$
$$\therefore\ 1248000 \times x = 20357568,$$
$$\therefore\ x = \frac{20357568}{1248000} = 16{\cdot}3 \text{ days.}$$

Ex. 7.

Let ABV represent a section of the pyramid; VD its perpendicular height; g its centre of gravity; and Abv its position when about to fall, its centre of gravity n being in the vertical line Ak.



From g draw gr parallel to AD, cutting Ak in r; then

$$\text{A}r = \text{D}g = \tfrac{1}{4} \text{ of DV} = \tfrac{1}{4} \text{ of } 12 = 3\,;$$
$$\text{AD} = \tfrac{1}{2} \text{ of AB} = \tfrac{1}{2} \text{ of } 8 = 4\,;$$
$$\text{A}n = \text{A}g = \sqrt{\text{AD}^2 + \text{D}g^2} = \sqrt{4^2+3^2} = 5\,;$$

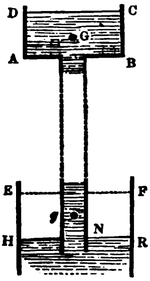
$rn = \text{A}n - \text{A}r = 5 - 3 = 2$ ft., which is the vertical distance through which the centre of gravity of the pyramid must be raised before it can be overturned.

Solidity of pyramid $= 8^2 \times \frac{12}{3}$,

$\therefore$ Weight of pyramid $= 8^2 \times \frac{12}{3} \times 100 = 25600$ lbs.

$\therefore$ Work in overturning the pyramid = weight $\times rn = 25600 \times 2 = 51200$.

Ex. 9.



Let A B C D represent the cistern; G the centre of gravity of the water when the cistern is full; H N R the level of the water in the well; then from the question,

A B the length of the cistern = 6 ft.; the breadth = 3 ft.; B C the depth = 4 ft.; and the height of A B from H R = 18 ft. Hence we have,

N G, or the vertical height through which the centre of gravity of the water is raised $= NB + \frac{1}{2} BC = 18 + 2 = 20$ feet.

Weight water $= 6 \times 4 \times 3 \times 62{\cdot}5 = 4500$ lbs.

$\therefore$ Work = wt. water $\times NG = 4500 \times 20 = 90000$.

In this example it is assumed that the water in the well always remains at the same level.

Additional Exercises.

Ex. 10.

Required the same as in Ex. 7., when the base of the pyramid is a rectangle whose length, A B, is 6 feet; breadth 4 feet; perpendicular height, D V, 16 feet; and the weight of a cubic foot of the material 120 lbs. (See fig. p. 17.)

Here, $Ar = Dg = \frac{1}{4}$ of $DV = \frac{1}{4}$ of $16 = 4$;

$AD = \frac{1}{2}$ of $AB = \frac{1}{2}$ of $6 = 3$;

$$A\,n = A\,g = \sqrt{A\,D^2 + D\,g^2} = \sqrt{3^2+4^2} = 5\,;$$

$r\,n = A\,n - A\,r = 5 - 4 = 1$ ft., which is the vertical distance though which the centre of gravity of the pyramid must be raised before it can be overturned.

Solidity of pyramid $= 6 \times 4 \times \frac{16}{3} = 128$ c. ft.

Wt. pyramid $= 128 \times 120 = 15360$ lbs.

$\therefore$ Work in overturning the pyramid = wt. in lbs. $\times\, r\,n$ $= 15360 \times 1 = 15360$.

Ex. 11.

Required the same as in Ex. 7., when the base of the pyramid is a circle whose diameter, A B, is 16 feet; perpendicular height, D V, is 24 feet; and the weight of a cubic foot of the material is 100 lbs.

Here $A\,r = D\,g = \frac{1}{4}$ of $24 = 6$ ft.;
$A\,D = \frac{1}{2}$ of $16 = 8$ ft.;
$A\,n = A\,g = \sqrt{8^2+6^2} = 10$ ft.;
$r\,n = A\,n - A\,r = 10 - 6 = 4$ ft.;

Solidity cone $= 16^2 \times \cdot 7854 \times \frac{24}{3} = 1608\cdot 4992$ c. ft.

Wt. cone in lbs. $= 1608\cdot 4992 \times 100 \doteq 160849\cdot 92$.

$\therefore$ Work in overturning the cone $= 160849\cdot 92 \times 4 = 643399\cdot 68$.

Ex. 12.

What must be the height of the pyramid in Ex. 10., when the work requisite for overturning the solid on the edge A is double the work requisite for overturning it on the other edge A B?

Let x = the perpendicular height D V; then,

$$A\,r = D\,g = \frac{1}{4} D\,V = \frac{x}{4}\,;\ A\,D = 3\,;$$

$$An=Ag=\sqrt{AD^2+Dg^2}=\sqrt{3^2+\left(\frac{x}{4}\right)^2}=\sqrt{9+\frac{x^2}{16}};$$

$$rn=An-Ar=\sqrt{9+\frac{x^2}{16}}-\frac{x}{4};$$

Solidity of the pyramid $=6\times4\times\frac{x}{3}=8\,x$

$\therefore$ Wt. pyramid $=8\,x\times120=960\,x$;

$\therefore$ Work in overturning the pyramid on the edge A $=960\,x\times\left(\sqrt{9+\frac{x^2}{16}}-\frac{x}{4}\right)$

Proceeding in the same manner, we find,

Work in overturning the pyramid on the edge A B $=960\,x\times\left(\sqrt{4+\frac{x^2}{16}}-\frac{x}{4}\right)$.

Hence we have, by the question,

$$960\,x\left(\sqrt{9+\frac{x^2}{16}}-\frac{x}{4}\right)=2\times960\,x\left(\sqrt{4+\frac{x^2}{16}}-\frac{x}{4}\right),$$

$$\therefore\ 2\sqrt{4+\frac{x^2}{16}}-\sqrt{9+\frac{x^2}{16}}=\frac{x}{4},$$

Squaring both sides, &c.,

$$4\sqrt{4\times\frac{x^2}{16}}+4\sqrt{9+\frac{x^2}{16}}=x^2+100,$$

$$\therefore\ (64+x^2)(144+x^2)=x^4+200\,x^2+10000,$$

$$\therefore\ 8\,x^2=784,$$

$$\therefore\ x^2=\frac{784}{8}=98,$$

$$\therefore\ x=\sqrt{98}=9{\cdot}8 \text{ feet nearly.}$$

Ex. 13.

Required the same as in Ex. 9., when the cistern is 10 feet long, 5 feet broad, and 8 feet deep; and the bottom of the cistern stands 16 feet from the water in the well.

Here, $NG=NB+\frac{1}{2}BC=16+\frac{1}{2}$ of $8=20$ ft.

Wt. water $= 10 \times 5 \times 8 \times 62{\cdot}5 = 25000$ lbs.

$\therefore$ Work in raising the water = wt. water $\times$ N G $= 25000 \times 20 = 500000$.

WORK IN EXCAVATIONS WHEN THE MATERIAL IS TRANSPORTED BY MEANS OF BARROWS, ETC.

ART. 13.

Ex. 2.

No. relays $= \frac{240}{120} = 2$.

Hence for each shoveller we must have 2 barrowmen; and as there are 4 pickmen to 3 shovellers, the number of pickmen will be 4 times the $\frac{1}{3}$ of the number of shovellers.

No. shovellers to do the work in 1 day $= \frac{50000}{500} = 100$.
„ barrowmen „ „ $= 2 \times 100 = 200$.
„ pickmen „ „ $= \frac{4}{3}$ of $100 = 133\frac{1}{3}$.

Now since the work is to be done in 20 days, we have,
No. shovellers to do the work in 20 days $= \frac{1}{20}$ of $100 = 5$.
„ barrowmen „ „ $= \frac{1}{20}$ of $200 = 10$.
„ of pickmen „ „ $= \frac{1}{20}$ of $133\frac{1}{3} = 6\frac{2}{3}$.

Ex. 5.

No. relays $= \frac{360}{120} = 3$.

Hence there must be 3 times the number of barrowmen that there are shovellers.

No. days' work of shovellers $= \frac{25000}{500} = 50$.
„ „ pickmen = 2 times the $\frac{1}{3}$ of $50 = 33\frac{1}{3}$.
„ „ barrowmen = 3 times $50 = 150$.

∴ Total days' work $= 50 + 33\frac{1}{3} + 150 = 233\frac{1}{3}$.

∴ Cost $= 233\frac{1}{3} \times$ 2*s.* 6*d.* = 29*l.* 3*s.* 4*d.*

Ex. 7.

Here there will be 12 planks of 12 feet long each in the ramp.

∴ Length of ramp = 12 ft. × 12 = 144 ft.

No. relays on the ramp $= \frac{144}{80} = 1\cdot8$.

No. relays on the level $= \frac{300}{120} = 2\frac{1}{2}$.

Total number of relays $= 1\cdot8 + 2\cdot5 = 4\cdot3$.

Days' work of shovellers $= \frac{70000}{500} = 140$.

" " barrowmen $= 140 \times 4\cdot3 = 602$.

" " pickmen $= 140 \times \frac{2}{3} = 93\frac{1}{3}$.

∴ Total number days' work $= 140 + 602 + 93\frac{1}{3} = 835\frac{1}{3}$.

∴ Cost $= 835\frac{1}{3} \times$ 2*s.* = 83*l.* 10*s.* 8*d.*

Ex. 9.

Time each journey $= \frac{2}{8} + \frac{2}{12} = \frac{5}{12}$ hours.

∴ No. journeys per day $= 10 \div \frac{5}{12} = 24$.

∴ No. c. ft. conveyed per day $= 24 \times 27 \times 24 = 15552$.

∴ No days to complete the transport $= \frac{1200000}{15552} = 77\cdot16$.

∴ No. shovellers $= \frac{15552}{500} = 31\cdot104$,

∴ No. pickmen $= \frac{2}{3}$ of $31\cdot104 = 20\cdot736$.

No. days' work of shovellers $= \frac{1200000}{500} = 2400$,

∴ " " pickmen $= \frac{2}{3}$ of $2400 = 1600$,

∴ Total no. days' work $= 2400 + 1600 = 4000$.

∴ Cost $= 4000 \times$ 3*s.* = 600*l.*

WORK IN OVERCOMING THE RESISTANCE OF FLUIDS.

ART. 14.

Ex. 5.

Speed of the train per min. $= \frac{50 \times 5280}{60} = 4400$ ft.

Resistance air $= \left(\frac{50}{10}\right)^2 \times 33 = 5^2 \times 33 = 25 \times 33 = 825$ lbs.

Work due to the resistance of the air per min. $= 825 \times 4400 = 3630000$.

Work engine per min. $= 140 \times 33000 = 4620000$.

$\therefore$ Work engine per min. which goes to overcome friction = work engine per min. − work due to the resistance of the air per min. $= 4620000 - 3630000 = 990000$.

Work due to friction per min. in moving 1 ton $= 8 \times 4400 = 35200$.

$\therefore$ No. tons $= \frac{\text{work which overcomes friction}}{\text{work friction in 1 ton}} = \frac{990000}{35200} =$ 28·1 tons.

Or thus.

Let $x =$ the number of tons in the train; then

Resistance of friction in lbs. $= 8x$,

$\therefore$ Work due to friction per min. $= 8x \times 4400 = 35200x$.

The work due to the resistance of the air, and the work of the engine, will be the same as given in the foregoing solution. Now we have

Work due to friction per min. + work due to the air per min. = work engine per min.

Hence we have, by substituting the values of these quantities,

$$35200x + 3630000 = 4620000,$$

$$\therefore\ x = \frac{4620000 - 3630000}{35200} = \frac{990000}{35200} = 28{\cdot}1 \text{ tons},$$

which is the same result as before found.

Ex. 6.

From the solution to the foregoing question, it appears that, after allowing for the work due to the resistance of the air, there are 990000 units of work of the engine remaining; this work must be expended in overcoming the other resistances. We first proceed to find the work due to friction and gravity in moving 1 ton.

Work due to friction per min. in moving 1 ton $= 8 \times 4400 = 35200$.

Rise of the rail in 4400 ft. $= \frac{4400}{1000} = 4{\cdot}4$ ft.

$\therefore$ Work due to gravity per min. in moving 1 ton $= 2240 \times 4{\cdot}4 = 9856$.

Now observing that this work is done in favour of the engine, we have

Work due to friction and gravity per min. in moving 1 ton = work friction − work gravity $= 35200 - 9856 = 25344$.

$$\therefore\ \text{No. tons} = \frac{990000}{25344} = 39.$$

Or thus.

Let x = the weight of the train in tons; then

Work due to friction per min. $= 8 \times x \times 4400 = 35200 \times x$.

Work due to gravity per min. $= x \times 2240 \times 4{\cdot}4 = 9856 \times x$.

And by the solution to Ex. 5. we have

Work due to the resistance of the air per min. $= 3630000$.

Now as the train descends the incline, we have

Total work due to all the resistances per min. $= 35200 \times x + 3630000 - 9856 \times x$.

Work engine per min. $= 140 \times 33000 = 4620000$.

But when the train moves uniformly, the work of the engine is equal to the work of the resistances,

$$\therefore\ 35200 \times x + 3630000 - 9856 \times x = 4620000,$$

$$\therefore\ 25344 \times x = 990000,$$

$$\therefore\ x = \frac{990000}{25344} = 39 \text{ tons}.$$

THE MODULUS OF A MACHINE.

Art. 15.

Ex. 5.

No. c. ft. water in the cistern $= 7 \times 5 \times 6 = 210$,

$\therefore$ Weight water in the cistern $= 210 \times 62{\cdot}5 = 13125$.

Vertical height to which the centre of gravity of the water is raised, or N G (see fig. p. 18.) $=$ N B $+$ B G $= 37 + \frac{1}{2}$ of $6 = 40$ feet.

$\therefore$ Work $= 13125 \times 40 = 525000$.

Effective work of the man per hour $= 2600 \times 60 \times {\cdot}66 = 102960$.

$$\therefore\ \text{No. hours to fill the cistern} = \frac{525000}{102960} = 5{\cdot}1 \text{ nearly}.$$

Ex. 8.

In fig. p. 18., let EF be the level of the water in the well at first; HNR the level after the tank ABCD is filled; *g* the centre of gravity of the water, HRFE, taken out of the well; and G the centre of gravity of the water in the tank.

No. c. ft. water in the tank $= 12 \times 9 \times 8 = 864$.

Now since the volume of the water taken out of the well is equal to the content of the tank, we have

FR, or the depth to which the water is depressed in the well $= \frac{\text{content tank}}{\text{section of well}} = \frac{864}{20} = 43{\cdot}2$ feet.

$\therefore$ Gg, or distance between the centres of gravity of the water in the tank and the same water in the well $= BF + \frac{1}{2} BC + \frac{1}{2} FR = 40 + \frac{1}{2}$ of $8 + \frac{1}{2}$ of $43{\cdot}2 = 40 + 4 + 21{\cdot}6 = 65{\cdot}6$ feet.

$\therefore$ Work in pumping water $=$ weight water $\times Gg = 864 \times 62{\cdot}5 \times 65{\cdot}6 = 3542400$.

Effective work of the engine per min. $= 4 \times 33000 \times {\cdot}66 = 87120$.

$\therefore$ No. min. to pump the water $=$

$$\frac{\text{work in pumping water}}{\text{effective work engine per min.}} = \frac{3542400}{87120} = 40{\cdot}6.$$

WORK OF WATER.

Work of a Water-wheel.

ART. 17.

Ex. 2.

Volume of water going over the fall per min. $= 3 \times 2 \times 12 = 72$ c. ft.

Weight of water going over the fall per min. $= 72 \times 62{\cdot}5$ lbs. $= 4500$ lbs.

$\therefore$ Work of the water per min. $= 4500 \times 20 = 90000$.

But the wheel does $\frac{7}{10}$ of this work,

$\therefore$ Work wheel per min. $= \frac{7}{10}$ of $90000 = 63000$.

$$\therefore \text{H. P.} = \frac{63000}{33000} = 1{\cdot}9 +$$

Allowing that a horse power is able to grind 1 bushel of corn per hour, we have

No. bushels per day $= 1{\cdot}9 \times 10 = 19$ bushels.

Ex. 5.

Let $x =$ no. c. ft. of water pumped per min.

No. c. ft. water going over the fall per min. $= 180 - x$.

$\therefore$ Weight of water " " $= (180 - x)\, 62{\cdot}5$.

$\therefore$ Effective work wheel per min. $= (180 - x)\, 62{\cdot}5 \times 20 \times {\cdot}7$.

Work in pumping water per min. $= x \times 62{\cdot}5 \times 40$.

$$\therefore x \times 62{\cdot}5 \times 40 = (180 - x)\, 62{\cdot}5 \times 20 \times \tfrac{7}{10};$$

$$x \times 2 = (180 - x)\, \tfrac{7}{10};$$

$$\therefore 20\, x = 1260 - 7\, x;$$

$$\therefore x = 46\tfrac{2}{3}, \text{ no. c. ft. pumped.}$$

Ex. 7.

Work of the wheel per min. $= 33000 \times 2 = 66000$.

Effective work of 1 c. ft. of water $= 62{\cdot}5 \times 20 \times {\cdot}68 = 850$.

$$\text{No. c. ft. water} = \frac{\text{work of wheel}}{\text{work of 1 c. ft. water}} = \frac{66000}{850} = 77{\cdot}6+$$

Or thus.

Let $x =$ the no. c. ft. of water;

$\therefore$ Wt. of water $= x \times 62{\cdot}5$;

$\therefore$ Work of water per min. $= x \times 62{\cdot}5 \times 20$;

$\therefore$ Work of the wheel per min. $= x \times 62{\cdot}5 \times 20 \times {\cdot}68 = x \times 850$.

But this work must be equivalent to 2 horse powers;

$$\therefore x \times 850 = 33000 \times 2.$$

$$\therefore x = \frac{33000 \times 2}{850} = 77{\cdot}6+$$

Ex. 8.

Here, the weight of the water going over the fall per min. $= 160 \times 62{\cdot}5 = 10000$ lbs.

Effective work of water per min. for 1 foot fall $= 10000 \times 1 \times {\cdot}68 = 6800$.

Work in the wheel per min. $= 33000 \times 3 = 99000$.

$$\therefore \text{No. feet in fall} = \frac{\text{work in wheel}}{\text{work in 1 foot fall}} = \frac{99000}{6800} = 14{\cdot}5+\text{ft.}$$

Or thus.

Let $x =$ no. feet in the fall; then,

Effective work water per min. $= 10000 \times x \times {\cdot}68 = 6800\,x$.

Work in the wheel per min. $= 99000$.

But the height of the fall is to be such as to produce this work;

$$\therefore 6800\,x = 99000;$$

$$\therefore x = \frac{99000}{6800} = 14{\cdot}5+\text{ft.}$$

Work of the Hydraulic Ram.

ART. 18.

Ex. 2.

Weight water spent per min. = no. gals. × wt. each gal. $= 11 \times 10 = 110$ lbs.

Effective work per min. $= 110 \times 9 \times {\cdot}87 = 861{\cdot}3$.

Work in raising 1 c. ft. of water $= 62{\cdot}5 \times 30 = 1875$.

$$\therefore \text{ No. c. ft. water} = \frac{861{\cdot}3}{1875} = {\cdot}46 \text{ nearly.}$$

Or thus.

Let x = no. cubic feet of water raised per min.;

$\therefore$ Work in raising this water $= 62{\cdot}5 \times x \times 30$.

But this must be equal to the effective work of the ram per minute;

$$\therefore\ 62{\cdot}5 \times x \times 30 = 861{\cdot}3,$$

$$\therefore\ x = \frac{861{\cdot}3}{62{\cdot}5 \times 30} = {\cdot}46.$$

Ex. 3.

Here, the effective work per min. $= 140 \times 10 \times {\cdot}87 = 1218$.

Work in raising $\frac{1}{2}$ ft. of water to the height of 1 ft. $= 62{\cdot}5 \times \frac{1}{2} \times 1 = 31{\cdot}25$.

$$\therefore \text{ Height in feet} = \frac{1218}{31{\cdot}25} = 39 \text{ feet.}$$

Or thus.

Let x = height to which the water is raised in feet.

$\therefore$ Work in raising the water per min. $= 62{\cdot}5 \times \frac{1}{2} \times x$.

But this is also the effective work of the ram;

$$\therefore\ 62{\cdot}5 \times \tfrac{1}{2} \times x = 1218,$$

$$\therefore\ x = \frac{1218 \times 2}{62{\cdot}5} = 39 \text{ feet.}$$

ADDITIONAL EXERCISES.

Ex. 4.

The fall of the water in a hydraulic ram is 20 feet, the water spent per minute 10 gallons; it is required to find the height to which the engine will raise 40 cubic feet of water per hour, the modulus being ·87.

Let x = the height in feet.

Work in raising the water per hour $= 62·5 \times 40 \times x = 2500 \times x$.

Wt. water spent per hour $= 10 \times 10 \times 60 = 6000$ lbs.

Effective work of the ram per hour $= 6000 \times 20 \times ·87 = 104400$.

$$\therefore 2500 \times x = 104400,$$

$$\therefore x = \frac{104400}{2500} = 41·6 + \text{ feet.}$$

Ex. 5.

In a hydraulic ram, the water spent per min. = 15 gallons, the fall of the water = 10 feet, the water raised per hour = 39 cubic feet, and the height to which this water is raised = 30 feet; required the modulus of the ram.

Let x = the modulus of the ram.

Work per hour in raising the water $= 39 \times 62·5 \times 30$.

Work water per hour $= 15 \times 10 \times 10 \times 60$.

Effective work of ram per hour $= 15 \times 10 \times 10 \times 60 \times x$.

$$\therefore 15 \times 10 \times 10 \times 60 \times x = 39 \times 62·5 \times 30;$$

$$\therefore x = \frac{39 \times 62·5 \times 30}{15 \times 10 \times 10 \times 60} = ·81.$$

WORK OF STEAM.

Work of Steam, having a mean Pressure.

Art. 22.

Ex. 3.

Here, load $+\frac{1}{7}$ load $+ 1 + 15 = 48$,

$\therefore$ $\frac{8}{7}$ load $= 32$, and load $= 28$.

Useful work engine per min. $28 \times 500 \times 6 \times 16 \times {\cdot}66 = 887040$.

Work in pumping 1 foot water $= 62{\cdot}5 \times 160 \times 6 = 60000$;

$$\therefore \text{ No. c. ft.} = \frac{887040}{60000} = 14{\cdot}7 +$$

Ex. 4.

Work done upon 1 inch of the piston in 1 stroke $= 33 \times 4 = 132$.

Work upon the whole piston in 1 stroke $= 132 \times 500 = 66000$.

Work upon the whole piston in 16 strokes $= 66000 \times 16 = 1056000$.

As this is also the work done per minute, we have,

$$\text{H. P.} = \frac{1056000}{33000} = 32.$$

Ex. 7.

Work per min. of steam with 1 lb. effective pressure $= 800 \times 6 \times 20 = 96000$.

Effective work per min. $= 33000 \times 40 = 1320000$;

$\therefore$ Effective pressure on 1 inch of piston $= \frac{1320000}{96000} = 13{\cdot}75$;

$\therefore$ Pressure steam $= 1 + 15 + 13{\cdot}75 + \frac{1}{7}$ of $13{\cdot}75 = 31{\cdot}7 +$ lbs.

Ex. 9.

Work done per min. $= 80 \times 62{\cdot}5 \times 100 \times 6 = 3000000$;

$\therefore$ Work engine per min. $= \frac{3000000}{{\cdot}66} = 4545454$;

Useful work on 1 inch piston in 1 stroke $= \frac{4545454}{20 \times 1000} = 227{\cdot}27$;

$\therefore$ Useful load $= \frac{227{\cdot}27}{10} = 22{\cdot}727$;

$\therefore$ Pressure steam $= 22{\cdot}727 + \frac{1}{7}$ of $22{\cdot}727 + 15 + 1 = 41{\cdot}9 +$ lbs.

Work of Steam, with a mean Pressure, considered in relation to the Water evaporated.

ART. 23.

Ex. 2.

Here, load $+ \frac{1}{7}$ load $+ 15 + 1 = 48$, $\therefore$ load $= 28$ lbs.
By the experimental table, Art. 23.,
Volume 1 c. ft. water in steam at 48 lbs. $= 575$ c. ft.
$\therefore$ Volume of steam evaporated per min. $= 575 \times {\cdot}4 = 230$ c. ft.

Volume discharged at each stroke $= \frac{120 \times 2\frac{1}{4}}{144} = 1{\cdot}875$;

$\therefore$ No. strokes per min. $= \frac{230}{1{\cdot}875} = 122{\cdot}6$.

Work in 1 stroke $= 28 \times 120 \times 2\frac{1}{4} = 7560$.
Work per min. $= 28 \times 120 \times 2\frac{1}{4} \times 122{\cdot}6 = 926856$;

$$\therefore \text{H. P.} = \frac{926856}{33000} = 28.$$

Ex. 4.

Useful load $+ \frac{1}{7}$ of useful load $+ 15 + 1 = 60$,
$\therefore$ Useful load $= 38\frac{1}{2}$ lbs.

Volume steam discharged per min. $=\frac{96\times 2\frac{1}{2}}{144}\times 100=166\frac{2}{3}$;

" " per hour $=166\frac{2}{3}\times 60=10000.$

By the experimental table,

Volume 1 c. ft. water in steam at 60 lbs. = 470;

$\therefore$ Volume steam evapo. per hour $=\frac{10000}{470}=21\cdot 2+$ c. ft.

$$\text{H. P.}=\frac{38\frac{1}{3}\times 2\frac{1}{2}\times 96\times 100}{33000}=28.$$

Ex. 7.

Total resistance to the motion of the carriages

$$=7\times 100+\left(\frac{40}{10}\right)^2\times 33=1228\text{ lbs.}$$

Space moved over in 1 revo. of the driving wheel = circum. wheel $=6\times 3\cdot 1416=18\cdot 8496$ feet;

$\therefore$ Work in 1 revo. $=18\cdot 8496\times 1228=23147\cdot 3.$

Now as the engine has two cylinders, and as each piston makes two strokes while the driving wheel turns round once, it follows that the piston will perform 4 single strokes while the driving wheel turns round once, hence we have,

Work of 1 lb. per inch pressure on the piston, in one revo. of the driving wheel $=1\times 120\times 1\cdot 5\times 4=720.$

Effective pressure on 1 inch of the piston

$$=\frac{\text{work in 1 revo.}}{\text{work of 1 lb. \&c.}}=\frac{23147}{720}=32\cdot 1\text{ lbs.}$$

Resistance due to the blast pipe $=1\cdot 75\times\frac{40}{10}=7$ lbs.

$\therefore$ Total pressure of steam on the piston $=32\cdot 1+\frac{1}{7}$ of $32\cdot 1+7+1+15=59\cdot 68$ lbs., or 60 lbs. nearly.

No. revo. driving wheel per min.

$$=\frac{\text{space moved over per min.}}{\text{circum. driving wheel}}=\frac{40\times 5280}{60\times 18\cdot 8496}=186\cdot 8.$$

$\therefore$ No. strokes of the piston per min. $=186\cdot 8\times 4=747\cdot 2$;

∴ Volume steam discharged per min. $=\frac{120}{144}\times 1{\cdot}5\times 747{\cdot}2$ $=934$ c. ft.

But from the experimental table, 1 foot of water produces 470 feet of steam at 60 lbs. pressure;

∴ No. c. feet water evapo. per min. $=\frac{934}{470}=1{\cdot}98$ c. ft.

Now a bushel of coals is supposed to evaporate 11·5 c. ft. of water;

∴ No. bushels of coals used per min. $=\frac{1{\cdot}98}{11{\cdot}5}$.

The no. minutes the engine is in going over 200 miles $=\frac{200\times 60}{40}=300$;

∴ No. bushels of coals used for 300 min. $=\frac{1{\cdot}98}{11{\cdot}5}\times 300=$ 51 bushels+.

Ex. 8.

Here the distance travelled $=200$ miles.

The no. bushels of coals consumed $=51$.

∴ No. bushels for 1 mile $=\frac{51}{200}$

∴ Cost per mile for 40 tons $=\frac{51\times 15}{200}$ pence.

∴ „ „ for 1 ton $=\frac{51\times 15}{200\times 40}$ pence $=\cdot 382$ farthings, or a little more than $\frac{1}{3}$ of a farthing.

Ex. 10.

Here at 60 lbs. pressure 1 c. ft. of water forms 470 cubic feet of stream (see table Art. 23.).

∴ Volume steam formed per min. $\doteq 470\times\cdot 7=329$ ft.

Volume cylinder $=\frac{90\times 16}{1728}$ c. ft.

∴ Volume steam discharged in one revolution of the driving wheel $=\frac{90\times 16}{1728}\times 4=3\frac{1}{3}$ c. f.

$\therefore$ No. revo. of wheel per min. $=\frac{329}{3\frac{1}{3}}$ (multiplying numerator and denominator by 3) $=\frac{987}{10}=98{\cdot}7$.

Space moved over by the carriage in 1 revo. of the driving wheel $=6\times3{\cdot}1416$ ft.

$\therefore$ Space moved over by the carriage per min. or in 98·7 revo. of the driving wheel $=6\times3{\cdot}1416\times98{\cdot}7$ ft.

$\therefore$ Space moved over per hour $=\frac{6\times3{\cdot}1416\times98{\cdot}7\times60}{5280}$ miles $=21{\cdot}1+$ miles.

Ex. 12.

(1.) To find the number of strokes of the piston.

Distance moved over by the train per min. $=\frac{50\times5280}{60}$ $=4400$ feet.

Distance moved over by the train in 1 revo. of the driving wheel $=5\times3{\cdot}1416$ feet $=15{\cdot}708$ feet.

$\therefore$ No. revo. of the wheel per min.

$$=\frac{\text{Distance moved per min.}}{\text{Distance in 1 revo. wheel}}=\frac{4400}{15{\cdot}708}=280.$$

$\therefore$ No. strokes of the piston per min. $=280\times4=1120$.

(2.) To find the useful load.

Load $+\frac{1}{7}$ load $+1+15+1{\cdot}75\times\frac{50}{10}=$ whole pressure of the steam, or 60 lbs.

$\therefore$ $\frac{8}{7}$ load $=60-1-15-1{\cdot}75\times5=35{\cdot}25$ lbs.

$\therefore$ Load $=\frac{35{\cdot}25\times7}{8}=30{\cdot}84$ lbs.

(3.) To find the effective work.

$\therefore$ Effective work in 1 stroke of the piston $=30{\cdot}84\times120\times\frac{5}{4}=4626$.

$\therefore$ Effective work engine per min. or in 1120 strokes $=4626\times1120=5181120$.

∴ Effective H. P. $= \frac{5181120}{33000} = 157.$

(4.) To find the weight of the train.

In this example the effective work just found has to sustain a speed of 50 miles per hour, or 4400 feet per minute, in opposition to the resistances of friction and the resistance of the air.

Work due to the resistance of the atmosphere per min. $= \left(\frac{50}{10}\right)^2 \times 33 \times 4400 = 3630000.$

∴ Work due to friction per min. = effective work engine per min. – work of the resistance of the air per min. = $5181120 - 3630000 = 1551120.$

Work of friction per min. when train is 1 ton $= 7 \times 4400 = 30800.$

∴ No. tons in the train $= \frac{\text{work friction per min.}}{\text{work friction per min. for 1 ton}}$ $= \frac{1551120}{30800} = 50{\cdot}3$ tons.

(5.) To find the water evaporated from the boiler.

Volume steam discharged each stroke $= \frac{120 \times \frac{5}{4}}{144} = \frac{150}{144}$ c. ft.

∴ Volume steam discharged per min. or in 1120 strokes $= \frac{150 \times 1120}{144} = 1166\frac{2}{3}.$

But, by the table, a cubic foot of water yields 470 cubic feet of steam at 60 lbs. pressure.

∴ No. c. ft. of water evaporated per min. $= \frac{1166\frac{2}{3}}{470} =$ 2·48 c. ft.

Ex. 13. p. 48.

(1.) To find the duty of the engine.

Here by Ex. 1. p. 42., the useful work of the engine per minute is 24×33000, or 24 H. P.

This work, it will be observed, is performed by the evaporation of ·335 c. ft. of water. Now the duty of an engine is the work which that engine will perform with a bushel of coals, which we assume to be sufficient to evaporate 11·5 c. ft. of water; hence we have,

Work of ·335 c. ft. water $= 24 \times 33000$.

∴ Work of 11·5 c. ft. water, or work done by 1 bushel of coals $= \dfrac{24 \times 33000 \times 11{\cdot}5}{{\cdot}335} = 27\frac{1}{2}$ millions nearly, which is the duty of the engine.

(2.) To find the cost of a H. P.

No. c. ft. water evapo. for 24 H. P. per min. = ·335.

" " for 1 H. P. in 6 h. $= \dfrac{{\cdot}335 \times 60 \times 6}{24}$ $= 5{\cdot}025$ c. ft.

But since 1 bushel of coals evaporates 11·5 cubic feet of water, we have

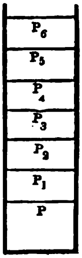
No. bushels of coals for 1 H. P. in 6 h. $= \dfrac{5{\cdot}025}{11{\cdot}5}$

∴ Cost of " " " $= \dfrac{5{\cdot}025}{11{\cdot}5} \times 4\text{d.} = 1\frac{1}{2}\text{d.}+$.

Work of Steam used expansively.

ART. 25.

Ex. 2.



Here the space through which the steam acts with a constant pressure is 2 ft., and therefore the space through which it acts expansively is 8 ft. less by 2 ft., or 6 ft. Dividing this space into six equal parts, each interval will be 1 foot. Putting P for the pressure of the steam on the piston at first, or during that part of the stroke where it acts with a constant pressure; P_1 the pressure of the steam on the piston at the end of the first interval; P_2 the pressure at the end of the second interval, and so on; then we have by Marriotte's law

P, or pressure at 2 ft. of the stroke = 48 lbs.

P_1, „ „ 3 ft. „ „ $=\frac{48\times 2}{3}=32$ lbs.

P_2, „ „ 4 ft. „ „ $=\frac{48\times 2}{4}=24$ lbs.

P_3, „ „ 5 ft. „ „ $=\frac{48\times 2}{5}=19{\cdot}2$ lbs.

P_4, „ „ 6 ft. „ „ $=\frac{48\times 2}{6}=16$ lbs.

P_5, „ „ 7 ft. „ „ $=\frac{48\times 2}{7}=13{\cdot}71$ lbs.

P_6, „ „ 8 ft. „ „ $=\frac{48\times 2}{8}=12$ lbs.

Now we have by Thomas Simpson's rule:

Work done expansively on 1 inch of the piston in 1 stroke

$$=\tfrac{1}{3}\{48+12+4(32+19{\cdot}2+13{\cdot}71)+2(24+16)\}$$

$$=\tfrac{1}{3}\{48+12+259{\cdot}64+80\}=133{\cdot}21.$$

Work done before the steam is cut off, that is, with a constant pressure of 48 lbs. $=48\times 2=96$.

Work done against the piston by the vapour in the condenser $=4\times 8=32$.

∴ Total work on 1 inch of the piston $=133{\cdot}2+96-32=197{\cdot}2$.

Ex. 3.

Here, work on 1 inch of the piston in 1 stroke $=197{\cdot}2$.

∴ Work on the whole piston in 1 stroke $=197{\cdot}2\times 2400$.

∴ Work on the whole piston in 10 strokes, or work per min. $=197{\cdot}2\times 2400\times 10$.

Work in raising 1 c. ft. of water $=62{\cdot}5\times 120\times 6$.

∴ No. c. ft. water raised per min. $=\frac{197{\cdot}2\times 2400\times 10}{62{\cdot}5\times 120\times 6}=105$.

Ex. 5.

Here the space through which the steam acts expansively is 8 feet, and dividing this space into 4 equal parts, the space of each interval will be 2 feet; hence we have, by Marriotte's law,

P, or pressure at 2 ft. of the stroke = 50 lbs.

P_1, „ „ 4 ft. „ „ $=\frac{50\times 2}{4}=25$ lbs.

P_2, „ „ 6 ft. „ „ $=\frac{50\times 2}{6}=16\frac{2}{3}$ lbs.

P_3, „ „ 8 ft. „ „ $=\frac{50\times 2}{8}=12\frac{1}{2}$ lbs.

P_4, „ „ 10 ft. „ „ $=\frac{50\times 2}{10}=10$ lbs.

Now we have by Thomas Simpson's rule:

Work done expansively on 1 inch of the piston in 1 stroke

$$=\tfrac{2}{3}\{50+10+4(25+12\tfrac{1}{2})+2\times 16\tfrac{2}{3}\}$$
$$=\tfrac{2}{3}\{60+150+33\tfrac{1}{3}\}=162{\cdot}2.$$

Work done before the steam is cut off $=50\times 2=100$.

∴ Total work steam on 1 inch $=162{\cdot}2+100=262{\cdot}2$.

∴ Mean pressure steam $=\frac{\text{work steam on 1 inch}}{\text{length of stroke}}=\frac{262{\cdot}2}{10}=26{\cdot}22$ lbs.

Resistances = load $+\frac{1}{7}$ load $+1+4$.

Now these resistances make up the mean pressure of the steam,

$$\therefore \text{Load}+\tfrac{1}{7}\text{ load}+1+4=26{\cdot}22.$$
$$\therefore \tfrac{8}{7}\text{ load}=21{\cdot}22,$$
$$\therefore \text{Load}=\frac{21{\cdot}22\times 7}{8}=18{\cdot}56\text{ lbs.}$$

At 50 lbs. pressure of steam, by the table, p. 42., one cubic foot of water gives rise to 554 cubic feet of steam.

∴ Volume steam discharged per min. $= \cdot 8 \times 554 =$ 443·2 c. ft.

Volume steam discharged at each stroke $= \frac{2000}{144} \times 2$ c. ft.

∴ No. strokes of piston per min.

$$= \frac{\text{vol. steam disc. per min.}}{\text{vol. steam disc. each stroke}} = 443{\cdot}2 \div \frac{2000}{144} \times 2 =$$

$$\frac{443{\cdot}2 \times 144}{2000 \times 2} = 15{\cdot}955.$$

Useful work per min. = load × area piston × length of stroke × no. strokes per min. $= 18{\cdot}56 \times 2000 \times 10 \times 15{\cdot}955.$

$$\therefore \text{Effective H. P.} = \frac{18{\cdot}56 \times 2000 \times 10 \times 15{\cdot}955}{33000} = 179{\cdot}2.$$

Ex. 6.

Here the space through which the steam acts expansively is $10 - 1\frac{2}{3}$ or $8\frac{1}{3}$ feet. Dividing this space into 4 equal parts, the interval between each will be equal to $\frac{1}{4}$ of $8\frac{1}{3}$ ft. or $2\frac{1}{12}$ ft.; hence we have by Marriotte's law,

$P = 60$ lbs.

$P_1 = \frac{60 \times 1\frac{2}{3}}{1\frac{2}{3} + 2\frac{1}{12}}$ (multiplying num. and deno. by 12) $=$ $\frac{60 \times 20}{20 + 25} = 26{\cdot}666$ lbs.

$$P_2 = \frac{60 \times 1\frac{2}{3}}{1\frac{2}{3} + 2\frac{1}{12} \times 2} = \frac{60 \times 10}{10 + 25} = 17{\cdot}14 \text{ lbs.}$$

$$P_3 = \frac{60 \times 1\frac{2}{3}}{1\frac{2}{3} + 2\frac{1}{12} \times 3} = \frac{60 \times 20}{20 + 75} = 12{\cdot}63 \text{ lbs.}$$

$$P_4 = \frac{60 \times 1\frac{2}{3}}{1\frac{2}{3} + 2\frac{1}{12} \times 4} = \frac{60 \times 1\frac{2}{3}}{10} = 10 \text{ lbs.}$$

By Thomas Simpson's rule, we have work done expansively on 1 inch of the piston in 1 stroke =

$$\frac{2\frac{1}{12}}{3}\{60 + 10 + 4(26{\cdot}666 + 12{\cdot}63) + 2 \times 17{\cdot}14\} = \frac{2\frac{1}{12}}{3} \times 261{\cdot}48$$

$= 181{\cdot}58.$

Work done before the steam is cut off $= 60 \times 1\frac{2}{3} = 100$.

$\therefore$ Total work steam on 1 inch $= 181{\cdot}58 + 100 = 281{\cdot}58$.

$\therefore$ Mean pressure steam $= \frac{281{\cdot}58}{10} = 28{\cdot}158$ lbs.

This mean pressure of the steam being equal to the sum of all the resistances on the piston, we have,

Load $+ \frac{1}{7}$ load $+ 1 + 4 = 28{\cdot}158$ lbs.

$\therefore$ Load $= \frac{23{\cdot}158 \times 7}{8} = 20{\cdot}2 +$ lbs.

Additional Exercises.

Ex. 9.

The length of the stroke of a condensing engine is 5 feet; the steam is cut off at 1 foot of the stroke: it is required to find the pressure of steam before expansion, when the work of the steam on each inch of the piston in one stroke $= 70$.

Let $x =$ the no. lbs. pressure of the steam on each inch. Dividing the space through which the steam acts expansively into 4 equal parts, the distance between each division will be 1 foot; hence we have by Marriotte's law,

P, or pressure at 1 ft. of the stroke $= x$ lbs.

P_1, " " 2 ft. " " $= \frac{x}{2}$ lbs.

P_2, " " 3 ft. " " $= \frac{x}{3}$ lbs.

P_3, " " 4 ft. " " $= \frac{x}{4}$ lbs.

P_4, " " 5 ft. " " $= \frac{x}{5}$ lbs.

Hence we have by Thomas Simpson's rule:

Work done expansively on 1 inch of the piston in 1 stroke =

$$\frac{1}{3}\left\{x+\frac{x}{5}+4\left(\frac{x}{2}+\frac{x}{4}\right)+2\times\frac{x}{3}\right\}$$

$$=x\times\frac{1}{3}\left\{1+\frac{1}{5}+\quad\left(\frac{1}{2}+\frac{1}{4}\right)+\frac{2}{3}\right\}$$

$$=x\times\frac{1}{3}\left\{4+\frac{1}{5}+\frac{2}{3}\right\}=x\times\frac{73}{45}.$$

Work done before the steam is cut off $=x\times 1=x$.

$\therefore$ Total work of steam on 1 inch $=x\times\frac{73}{45}+x=x\times\frac{118}{45}$

But by the question this work is to be equal to 70 units,

$$\therefore\ x\times\frac{118}{45}=70,$$

$$\therefore\ x=\frac{70\times 45}{118}=26{\cdot}7 \text{ lbs. nearly.}$$

To find the point of the stroke at which the steam must be cut off so as to perform the greatest amount of work.

ART. 26.

Ex. 2.

Here the useless resistances $=1+4+\frac{1}{7}$ of $9{\cdot}1=6{\cdot}3$ lbs. This is the pressure of the steam at the end of the stroke, so as to yield all its work. Let $x=$ the point of the stroke where the steam must be cut off so as to have this pressure at the end of the stroke; then we have by Marriotte's law,

$$30\times x=6{\cdot}3\times 5,$$

$$\therefore\ x=\frac{6{\cdot}3\times 5}{30}=1{\cdot}05\text{ ft.}$$

Ex. 3.

Here the pressure of the steam at the end of the stroke must be 5 lbs.; hence we have, adopting the notation of the last example,

$$x \times 40 = 10 \times 5,$$

$$\therefore\ x = \frac{50}{40} = \frac{5}{4} \text{ feet.}$$

Ex. 4.

Here, $x \times 36 = 8 \times (3+2)$,

$$\therefore\ x = \frac{40}{36} = \frac{10}{9} = 1\tfrac{1}{9} \text{ feet.}$$

TRANSMISSION OF WORK BY SIMPLE MACHINES.

The Lever.

ART. 28.

Ex. 9.

Here the weight or W acts at the distance of 3 inches from the fulcrum, and the power or P at the distance of 6 feet or 72 inches from the fulcrum; hence we have,

$$W \times 3 = 60 \times 72,$$

$$\therefore\ W = \frac{60 \times 72}{3} = 1440 \text{ lbs.}$$

Ex. 11.

Here, 5 ft. = 60 in., and 4 cwts. = 448 lbs.

$$\therefore\ P \times 60 = 448 \times 2,$$

$$\therefore\ P = \frac{448 \times 2}{60} = 14{\cdot}9 + \text{ lbs.}$$

Ex. 12.

Here, $W = 5 \times 112 = 560$ lbs., the distance at which W acts from the fulcrum = 4 inches, and P = 80 lbs.; required the distance of P from the fulcrum.

$$\therefore\ 80 \times \text{ distance of } P = 560 \times 4,$$

$$\therefore\ \text{distance of } P = \frac{560 \times 4}{80} = 28 \text{ inches.}$$

Ex. 13.

Here, in the lever P A (see *fig.* page 59.), the pressure produced at $A = \frac{2 \times 12}{4} \times 8 = 48$ lbs.

In the lever A B, the pressure produced at $B = \frac{3 \times 12}{6} \times 48$ lbs. $= 288$ lbs.

In the lever B W, the pressure produced at $W = \frac{5 \times 12}{2} \times 288$ lbs. $= 8640$ lbs. $= 3{\cdot}8 +$ tons.

Ex. 14.

Let $x =$ the distance of the pressure, 3 lbs., from the fulcrum, then

$$3 \times x = 4 \times 3 + 7 \times 5,$$
$$\therefore x = \frac{47}{3} = 15\tfrac{2}{3} \text{ inches.}$$

The Lever when its weight is taken into account.

ART. 30.

Ex. 2.

Here, referring to *fig.* 1. at page 57., we have,

$$P \times A\,F + \text{wt. lever} \times C\,F = W \times B\,F.$$

Now $A\,F = 20$ in.; $C\,F = S\,F - S\,C = S\,F - \frac{1}{2}$ of $S\,R = 24 - \frac{1}{2}$ of $30 = 9$ in.; $B\,F = 4$ in.; $W = 92$ lbs.; weight lever $= 2$ lbs.; and P is required. By substituting these values in the general equation above given, we have,

$$P \times 20 + 2 \times 9 = 92 \times 4,$$
$$\therefore P \times 20 + 18 = 368,$$
$$\therefore P \times 20 = 350,$$
$$\therefore P = \frac{350}{20} = 17{\cdot}5 \text{ lbs.}$$

Ex. 3.

In this case the distance at which the power P acts from the fulcrum is 60 inches,

$$\therefore\ P \times 60 + 4 \times 26 = 168 \times 3,$$

$$\therefore\ P = \frac{400}{60} = 6\tfrac{2}{3} \text{ lbs.}$$

Ex. 4.

Here all the data are the same as in Ex. 1., excepting that $P = 10$ lbs., and its distance, x, from the fulcrum is required; hence we have,

$$10 \times x + 4 \times 26 = 168 \times 3,$$

$$\therefore\ x = \frac{400}{10} = 40 \text{ inches.}$$

Ex. 6.

Here, we have (see *fig.* 2. page 57.),

$$W \times BF = P \times AF + \text{wt. lever} \times CF.$$

Now $BF = 5$ in.; $AF = 13$ in.; $CF = SF - SC = SF - \frac{1}{2}$ of $SR = 14 - \frac{1}{2}$ of $18 = 5$ in.; $P = 50$ lbs.; wt. lever $=$ 2 lbs.; and W is required;

$$\therefore\ W \times 5 = 50 \times 13 + 2 \times 5,$$

$$\therefore\ W = \frac{640}{5} = 128 \text{ lbs.}$$

Ex. 7.

In this case W is given $= 150$ lbs., and its distance, x, from the fulcrum is required,

$$\therefore\ 150 \times x = 50 \times 13 + 2 \times 5,$$

$$\therefore\ x = \frac{640}{150} = 4{\cdot}26 \text{ inches.}$$

Ex. 8.

Let $x =$ the distance of the pressure, 30 lbs., from the fulcrum.

Distance of the centre of gravity of the lever from the fulcrum $= \frac{1}{2}$ of $36 - 4 = 14$ inches; then we have

$$30 \times x + 16 \times 14 = 2 \times 112 \times 4,$$

$$\therefore\ x = \frac{672}{30} = 22{\cdot}4 \text{ inches.}$$

Ex. 10.

In this case, we have (see *fig.* 2. p. 57.)

Pressure on $A \times AF = W \times BF +$ wt. lever $\times CF$.

But, $AF = 30$ ft.; $BF = 10$ ft.; $FR = 2$ ft.; $SR = 50$ ft.; $\therefore CF = CR - FR = \frac{1}{2}$ of $SR - FR = \frac{1}{2}$ of $50 - 2 = 23$ ft.; $W = 12$ cwts.; and wt. lever $= 2$ cwts.;

$$\therefore \text{Pressure on } A \times 30 = 12 \times 10 + 2 \times 23\ ;$$

$$\therefore \text{Pressure on } A = \frac{166}{30} = 5{\cdot}53 \text{ cwts.}$$

Now the whole pressure on the two props is equal to 12 cwts. + 2 cwts., or 14 cwts.;

$$\begin{aligned}\therefore \text{Pressure on } F &= \text{whole pressure} - \text{pressure on } A\\ &= 14 \text{ cwts.} - 5{\cdot}53 \text{ cwts.}\\ &= 8{\cdot}47 \text{ cwts.}\end{aligned}$$

ART. 31.

Ex. 3.

Here $W = 27{\cdot}1$ lbs. and its distance, x, from the centre F of the rod is required; first, when the pressure of the steam in the boiler is 40 lbs., and second, when the pressure is 20 lbs.

(First.) Total pressure of the steam on the valve = pressure steam per sq. in. × no. sq. in. in the valve $= 40 \times 6 = 240$ lbs.

$\therefore$ Effective pressure of steam on the lever = total pressure steam on the valve − wt. valve $= 240 - 4 = 236$ lbs.

$$\therefore 27{\cdot}1 \times x + 5 \times 10 = 236 \times 2,$$

$$\therefore x = \frac{422}{27{\cdot}1} = 15{\cdot}5 \text{ inches.}$$

(Second.) Proceeding in the same manner, we have effective pressure of steam on the lever $= 20 \times 6 - 4 = 116$ lbs.,

$$\therefore 27{\cdot}1 \times x + 5 \times 10 = 116 \times 2.$$

$$\therefore x = \frac{182}{27{\cdot}1} = 6{\cdot}7 \text{ inches.}$$

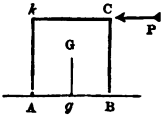
ART. 32.

Ex. 2.

$$\text{Here, advantage} = \frac{\text{length arm by which the power act}}{\text{length arm by which the resistance ac}}$$

$$= \frac{9}{1\frac{1}{2}} = 6.$$

Ex. 4.



Let A B Ck represent a section of the cube ; Gg a vertical line passing through its centre of gravity G ; P the pressure perpendicularly applied at C, tending to overturn the solid on A as a centre.

Here the leverage of P is $Ak = BC = 4$; the leverage the mass is $Ag = \frac{1}{2} AB = \frac{1}{2}$ of $4 = 2$; and the weight of t mass $ABCk = 4 \times 4 \times 4 \times 100 = 6400$ lbs. ;

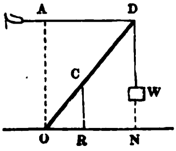
$$\therefore P \times Ak = \text{wt. mass} \times Ag,$$
$$\therefore P \times 4 = 6400 \times 2,$$
$$\therefore P = 3200 \text{ lbs.}$$

Ex. 6.

Here the tension of the cord = 30 cwts. ; W is require and as in Ex. 5., $OP = 15{\cdot}2$; $ON = 12{\cdot}5$; $OR = 6{\cdot}2$; a wt. pole = 2 cwts. ;

$$\therefore \text{Tension cord} \times OP = W \times ON + \text{wt. pole} \times OR;$$
$$\therefore 30 \times 15{\cdot}2 = W \times 12{\cdot}5 + 2 \times 6{\cdot}2,$$
$$\therefore W = \frac{30 \times 15{\cdot}2 - 2 \times 6{\cdot}2}{12{\cdot}5} = \frac{443{\cdot}6}{12{\cdot}5} = 35{\cdot}4 \text{ cwts.}$$

Ex. 7.



Here $OD = 50$; $DN = 40$; $W =$ 15 cwts. ; and wt. $OD = 1$ cwt. ; and the tension of the cord A D, acting in a horizontal direction, is required.

O A, the perpendicular upon A D, $= DN = 40$;

$$ON = \sqrt{OD^2 - DN^2} = \sqrt{50^2 - 40^2} = 30;$$

and because $OC = \frac{1}{2}$ of OD, $\therefore$ $OR = \frac{1}{2}ON = 15$; then by the equality of moments,

$$\text{Tension cord} \times OA = W \times ON + \text{wt. pole} \times OR,$$

$$\therefore \text{Tension cord} \times 40 = 15 \times 30 + 1 \times 15,$$

$$\therefore \text{Tension cord} = \frac{465}{40} = 11\cdot6+, \text{cwts.}$$

Ex. 8.

Here, $OD = 50$; $OC = 12$; $W = 15$ cwts.; wt. pole = 1 cwt.; tension cord = 12 cwts.; and ON is required. (See the last figure.)

Let $x = ON$; then by the similar triangles OND and ORC, we have

$$\frac{OR}{OC} = \frac{ON}{OD}, \text{ or}$$

$$\frac{OR}{12} = \frac{x}{50}, \therefore OR = \frac{12x}{50};$$

$$OA = ND = \sqrt{OD^2 - ON^2} = \sqrt{2500 - x^2}.$$

By the equality of moments, we now have

$$\text{Tension cord} \times OA = W \times ON + \text{wt. pole} \times OR,$$

$$\therefore 12 \times \sqrt{2500 - x^2} = 15 \times x + 1 \times \frac{12x}{50},$$

$$\therefore \sqrt{2500 - x^2} = \left(\frac{5}{4} + \frac{1}{50}\right)x,$$

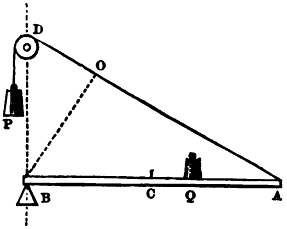
$$\therefore 2500 - x^2 = \left(\frac{127}{100}\right)^2 x^2,$$

$$\therefore x = 30\cdot9 \text{ feet.}$$

Ex. 10.

Here we have, as in Ex. 9.,

$$\text{Tension chain} \times BO = \text{wt. } Q \times QB + \text{wt. platform} \times CB.$$



Supposing the chain to go over a pulley at D, and a weight P attached to it, so as to support the platform; then the weight of P will be the tension of the chain, or the force with which it is stretched.

In this example, A B = 3 ft.; Q B = 2 ft.; B D = 4 ft.; wt. Q = 8 cwts.; wt. platform = 1 cwt.; and C B, the distance of the centre of gravity of the platform from B = $\frac{1}{2}$ of A B = $\frac{1}{2}$ of 3 = $\frac{3}{2}$ ft.

To find B O, we have

$$AD = \sqrt{AB^2 + BD^2} = \sqrt{3^2 + 4^2} = 5 \text{ ft.};$$

Area triangle A B D = $\frac{1}{2}$ A B × B D = $\frac{1}{2}$ of 3 × 4 = 6;

also, Area triangle A B D = $\frac{1}{2}$ A D × B O = $\frac{1}{2}$ of 5 × B O

$$= \frac{5}{2} BO;$$

$$\therefore \frac{5}{2} BO = 6,$$

$$\therefore BO = \frac{6 \times 2}{5} = 2{\cdot}4 \text{ ft.}$$

Substituting these values in the general equation, we have

$$\text{Tension chain } \times 2{\cdot}4 = 8 \times 2 + 1 \times \frac{3}{2},$$

$$\therefore \text{ Tension chain } = \frac{8 \times 2 + 1 \times \frac{3}{2}}{2{\cdot}4} = 7{\cdot}29 \text{ cwts.}$$

Ex. 11.

By the equality of moments, (see figure, page 65. text.)

Tension cord × O P = W × O N + wt. pole × O R.

(1.) To find O P.

By the common rule of mensuration for finding the area of a triangle, having given the three sides,

$$\text{Area triangle } A\,O\,D = 95 \text{ sq. ft.};$$

but we have also,

$$\begin{aligned}\text{Area triangle } A\,O\,D &= \tfrac{1}{2}\,A\,D \times O\,P \\ &= \tfrac{1}{2} \text{ of } 25 \times O\,P = 12{\cdot}5 \times O\,P,\end{aligned}$$

$$\therefore\ 12{\cdot}5 \times O\,P = 95,$$

$$\therefore\ O\,P = \frac{95}{12{\cdot}5} = 7{\cdot}6 \text{ ft.}$$

(2.) To find O N and O R.

In like manner, we have,

$$\begin{aligned}\text{Area triangle } A\,O\,D &= \tfrac{1}{2}\,A\,O \times D\,N \\ &= \tfrac{1}{2} \text{ of } 10 \times D\,N = 5 \times D\,N;\end{aligned}$$

$$\therefore\ 5 \times D\,N = 95,$$

$$\therefore\ D\,N = \frac{95}{5} = 19 \text{ ft.}$$

$$O\,N = \sqrt{O\,D^2 - D\,N^2} = \sqrt{20^2 - 19^2} = 6{\cdot}25 \text{ ft.}$$

$$\therefore\ O\,R = \tfrac{1}{2}\,O\,N = \tfrac{1}{2} \text{ of } 6{\cdot}25 = 3{\cdot}12 \text{ ft.}$$

Substituting these values in the general equation,

$$\begin{aligned}\text{Tension cord } \times 7{\cdot}6 &= 20 \times 6{\cdot}25 + 2 \times 3{\cdot}12 \\ &= 131{\cdot}26,\end{aligned}$$

$$\therefore\ \text{Tension cord} = \frac{131{\cdot}26}{7{\cdot}6} = 17{\cdot}3 \text{ cwts. nearly.}$$

Centre of Gravity.

Art. 33.

Ex. 2.

(Wt. A + wt. B + wt. C) × G F = wt. A × A F + wt. B × B F + wt. C × C F;

Now A F = 1; B F = A F + A B = 1 + 3 = 4; C F = A F + A B + B C = 1 + 3 + 10 = 14; wt. A = 2 lbs.; wt. B = 4 lbs.; and wt. C = 8 lbs.;

$$\therefore\ (2+4+8) \times G\,F = 2 \times 1 + 4 \times 4 + 8 \times 14,$$

$$\therefore\ G\,F = \frac{130}{14} = 9{\cdot}2+,$$

which is the distance of the centre of gravity from the assumed point F.

Ex. 3.

Let F be the centre of gravity of the two bodies; then they would balance each other on a fulcrum at F. Let $x = PF$; then $WF = 20 - x$; and by the equality of moments,

$$P \times PF = W \times WF; \text{ that is,}$$
$$2 \times x = 3 \times (20 - x),$$
$$\therefore\ 2x = 60 - 3x,$$
$$\therefore\ 5x = 60, \text{ and } x = 12,$$

which is the distance of the centre of gravity from P.

Art. 37.

Ex. 2.

Here we have,

$$(1+2+3)x = 1 \times 3 + 2 \times 4 + 3 \times 7;\ \therefore\ x = 5\tfrac{1}{3} \text{ ft.};$$
$$(1+2+3)y = 1 \times 2 + 2 \times 9 + 3 \times 10;\ \therefore\ y = 8\tfrac{1}{3} \text{ ft.}$$

Wheel and Axle.

Art. 38.

Ex. 2.

Here, W × radius axle = P × length handle; now in this example, W = 500 lbs.; P = 80 lbs.; length handle = 2 ft. = 24 in.;

$$500 \times \text{radius axle} = 80 \times 24,$$
$$\therefore \text{ radius axle} = \frac{80 \times 24}{500} = 3{\cdot}84 \text{ in.},$$
$$\therefore \text{ diam. axle} = 3{\cdot}84 \text{ in.} \times 2 = 7{\cdot}68 \text{ in. or } {\cdot}64 \text{ ft.}$$

Ex. 3.

Work P in 1 revo. $= 60 \times 4 \times 3{\cdot}1416$,

„ W „ $= \mathrm{W} \times \frac{6}{12} \times 3{\cdot}1416$,

$$\therefore\ \mathrm{W} \times \frac{6}{12} \times 3{\cdot}1416 = 60 \times 4 \times 3{\cdot}1416,$$

$$\therefore\ \mathrm{W} \times \frac{6}{12} = 60 \times 4,$$

$$\therefore\ \mathrm{W} = 480\ \text{lbs.}$$

Ex. 4.

Here, advantage $= \frac{\mathrm{W}}{\mathrm{P}} = \frac{480}{60} = 8$.

Ex. 5.

Here if $\frac{1}{7}$ of the power is destroyed by friction $\frac{6}{7}$ must remain; in this case, therefore, $\mathrm{W} = \frac{6}{7}$ of $480 = 411\frac{3}{7}$ lbs.

Cogged or Toothed Wheels.

Ex. 2.

Let the axle I and wheel Q make one revolution; then the no. revo. of the wheel C $= \frac{30}{6} = 5$;

$\therefore$ Space moved over by $\mathrm{W} = \frac{1}{3} \times 3{\cdot}1416$;

„ „ „ $\mathrm{P} = 2{\cdot}8 \times 3{\cdot}1416 \times 5$;

Work due to $\mathrm{W} = \frac{1}{3} \times 3{\cdot}1416 \times \mathrm{W}$;

„ „ $\mathrm{P} = 2{\cdot}8 \times 3{\cdot}1416 \times 5 \times 60$;

$$\therefore\ \tfrac{1}{3} \times 3{\cdot}1416 \times \mathrm{W} = 2{\cdot}8 \times 3{\cdot}1416 \times 5 \times 60;$$

$$\therefore\ \tfrac{1}{3}\mathrm{W} = 2{\cdot}8 \times 5 \times 60;$$

$$\therefore\ \mathrm{W} = 2{\cdot}8 \times 5 \times 60 \times 3 = 2520\ \text{lbs.}$$

Ex. 4.

Let $x =$ the no. teeth in the wheel Q.

Let the wheel Q and its axle make one revolution, then we have

No. revo. wheel $C = \frac{x}{6}$;

Space moved over by $P = 2·8 \times 3·1416 \times \frac{x}{6}$;

$$\therefore \text{ Work due to } P = 2·8 \times 3·1416 \times \frac{x}{6} \times 60;$$

$$\text{,,} \quad \text{,,} \quad W = \tfrac{1}{3} \times 3·1416 \times 2240 \times 2;$$

$$\therefore 2·8 \times 3·1416 \times \frac{x}{6} \times 60 = \tfrac{1}{3} \times 3·1416 \times 2240 \times 2;$$

$$\therefore 28 \times x = \tfrac{1}{3} \times 2240 \times 2;$$

$$\therefore x = \frac{2240 \times 2}{3 \times 28} = 53 + .$$

Compound Wheel and Axle.

Art. 39.

Ex. 3.

Let the handle H make one revolution.

Space moved over by $P = \frac{20 \times 2}{12} \times 3·1416$ ft.

Work of $P = \frac{20 \times 2}{12} \times 3·1416 \times 1 = \frac{10}{3} \times 3·1416.$

Space moved over by W in ft. $= \frac{1}{2}$ of {circum. A − circum. C} $= \frac{\frac{1}{2} \times 3·1416 - \frac{1}{3} \times 3·1416}{2} = \frac{1}{4} \times 3·1416 - \frac{1}{6} \times 3·1416$ $= (\frac{1}{4} - \frac{1}{6})3·1416 = \frac{1}{12} \times 3·1416.$

$$\therefore \text{ Work } W = \tfrac{1}{12} \times 3·1416 \times W;$$

$$\therefore \tfrac{1}{12} \times 3·1416 \times W = \tfrac{10}{3} \times 3·1416;$$

$$\therefore \tfrac{1}{12} W = \tfrac{10}{3}; \quad \therefore W = 40 \text{ lbs.}$$

Ex. 4.

$$\text{Work of W in 1 revo.} = \frac{\cdot 6 \times 3\cdot 1416 - \cdot 4 \times 3\cdot 1416}{2} \times 600$$

$$= \cdot 1 \times 3\cdot 1416 \times 600;$$

Work of P in 1 revo. $= 3 \times 3\cdot 1416 \times P$;

$$\therefore\ 3 \times 3\cdot 1416 \times P = \cdot 1 \times 3\cdot 1416 \times 600,$$

$$\therefore\ 3P = 60, \text{ and } P = 20 \text{ lbs.}$$

Ex. 5.

Here, proceeding as in Ex. 2., we have

P × length handle $+\frac{1}{2}$ W × radius small axle $=\frac{1}{2}$ W × radius large axle. Now P = 1 lb.; W = 160 lbs.; length handle = 20 in.; radius small axle = 2 in.; and radius large axle is required; hence we have

$$1 \times 20 + \tfrac{1}{2} \text{ of } 160 \times 2 = \tfrac{1}{2} \text{ of } 160 \times \text{radius large axle};$$

$$\therefore\ 180 = 80 \times \text{radius large axle};$$

$$\therefore\ \text{radius large axle} = \frac{180}{80} = 2\tfrac{1}{4} \text{ inches.}$$

The Pulley.

Art. 41.

Ex. 3.

In this case, while W ascends 1 foot, P descends 8 feet. If P descend 8 feet, the first moveable pulley will ascend $\frac{1}{2}$ of 8 feet, or 4 feet; the second $\frac{1}{2}$ of 4 feet or 2 feet; and the third $\frac{1}{2}$ of 2 feet or 1 foot.

Work of P $= 100 \times 8$;

Work of the 3rd moveable pulley $= 3 \times 1 = 3$,

„ „ 2d „ „ $= 3 \times 2 = 6$,

„ „ 1st „ „ $= 3 \times 4 = 12$

∴ Total work in raising the pulleys $= 3 + 6 + 12 = 21$;

Work of $W = W \times 1 = W$,

$\therefore$ Work of $P =$ work of $W +$ total work in raising the pulleys.

$$\therefore 100 \times 8 = W + 21\,;$$
$$\therefore W = 769 \text{ lbs.}$$

Ex. 5.

Here, as in Ex. 4., if W with its pulley B ascend 1 foot, A will ascend 3 feet, and P will descend 9 feet; hence we have

Work $P =$ work $W +$ work pulley $B +$ work pulley A;

$$\therefore P \times 9 = 600 \times 1 + 6 \times 1 + 2 \times 3\,;$$
$$\therefore P = \frac{612}{9} = 68 \text{ lbs.}$$

Ex. 6.

As A has 3 times the motion of B, so in like manner B will have 3 times the motion of the next pulley; if therefore the third pulley ascend 1 foot, B will ascend 3 feet, A will ascend 3 times 3 feet or 9 feet, and P will descend 3 times 9 feet or 27 feet. Hence 1 lb. at P will obviously balance 27 lbs. at W.

Inclined Plane.

Art. 42.

Ex. 2.

Here in order to raise W over 20 feet in opposition to gravity, P must descend 100 feet;

$$\therefore \text{Work in raising } W = 40 \times 20,$$

Work of $P = P \times 100$,

$$\therefore P \times 100 = 40 \times 20, \quad \therefore P = 8 \text{ lbs.}$$

Ex. 4.

Here, as the inclination of the plane is small, we have

Resistance of friction $= \frac{1}{280}$ of 56 lbs. $= \frac{2}{10}$ lb.;

$\therefore$ Work due to friction $= \cdot 2 \times 100 = 20$.
Work due to gravity $= 56 \times 2 = 112$;
Work of the pressure $P = P \times 100$;
$\therefore P \times 100 = 20 + 112, \therefore P = 1{\cdot}32$ lbs.

Ex. 6.

Resistance of friction $= \frac{1}{30}$ of 90 lbs. = 3 lbs.
$\therefore$ Work due to friction $= 3 \times 100 = 300$.
Work due to gravity $= 90 \times \cdot 2 = 18$.
Work of the pressure $P = P \times 100$.

Now as the body moves *down* the plane,

$$\therefore P \times 100 = 300 - 18;$$

$$\therefore P = \frac{282}{100} = 2{\cdot}82 \text{ lbs.}$$

Ex. 8.

Let x = the height of plane in feet;

$\therefore$ Work due to gravity $= 90 \times x$;

And work due to friction = 300, as in Ex. 6.; but in this case the work due to gravity must be equal to that of friction;

$$\therefore 90 \times x = 300; \therefore x = \frac{300}{90} = 3\tfrac{1}{3} \text{ ft.}$$

The moveable inclined Plane, or Wedge.

ART. 43.

Ex. 2.

Work applied for the whole length of the wedge $= 400 \times 2$
Work done $= W \times \cdot 4$;

$$\therefore W \times \cdot 4 = 400 \times 2,$$

$$\therefore W = \frac{800}{\cdot 4} = 2000 \text{ lbs.}$$

The Screw.

ART. 44.

Ex. 2.

Let the lever be turned round once; then

Space moved over by $P = 4 \times 3{\cdot}1416 = 12{\cdot}5664$;

" " " $W =$ thickness thread $= \frac{1}{24}$ ft.;

$\therefore$ Work of $P = 60 \times 12{\cdot}5664 = 753{\cdot}984$;

and work of $W = W \times \frac{1}{24}$;

$\therefore W \times \frac{1}{24} = 753{\cdot}984$;

$\therefore W = 753{\cdot}984 \times 24 = 18095$ lbs. +.

Ex. 3.

$$\text{Advantage} = \frac{W}{P} = \frac{18095}{60} = 301{\cdot}5+.$$

Ex. 5.

Work of P in 1 revo. $= P \times 3 \times 3{\cdot}1416$;

" W " $= 10 \times 2240 \times \frac{1}{12}$;

$\therefore P \times 3 \times 3{\cdot}1416 = 10 \times 2240 \times \frac{1}{12}$;

$$\therefore P = \frac{10 \times 2240}{3 \times 3{\cdot}1416 \times 12} = 198 \text{ lbs.}$$

The Compound Screw.

ART. 45.

Ex. 2.

Let the lever be turned round once; then

Space moved over by the press board $= \frac{7}{8} - \frac{3}{4} = \frac{1}{8}$ in. $= \frac{1}{96}$ ft.;

Work resistance, or work of $W = W \times \frac{1}{96}$;

Work applied $= 1 \times 2 \times 1{\cdot}5 \times 3{\cdot}1416 = 9{\cdot}4248$;

$\therefore W \times \frac{1}{96} = 9{\cdot}4248$;

$\therefore W = 9{\cdot}4248 \times 96 = 904$ lbs. +.

Ex. 3.

$$\text{Work of } P = P \times 3 \times 3{\cdot}1416;$$

$$\text{,, } W = 4 \times 2240 \times \frac{1}{48};$$

$$\therefore P \times 3 \times 3{\cdot}1416 = 4 \times 2240 \times \frac{1}{48};$$

$$P = \frac{4 \times 2240}{3 \times 3{\cdot}1416 \times 48} = 19{\cdot}8 \text{ lbs.}$$

The Endless Screw.

Art. 46.

Ex. 2.

Let the axle turn round once; then the handle must be turned round 20 times;

$$\therefore \text{Work applied} = 60 \times 1{\cdot}5 \times 2 \times 3{\cdot}1416 \times 20;$$

$$\text{and, Work done} = W \times {\cdot}3 \times 2 \times 3{\cdot}1416;$$

$$\therefore W \times {\cdot}3 \times 2 \times 3{\cdot}1416 = 60 \times 1{\cdot}5 \times 2 \times 3{\cdot}1416 \times 20;$$

$$\therefore W \times {\cdot}3 = 60 \times 1{\cdot}5 \times 20;$$

$$\therefore W = \frac{60 \times 1{\cdot}5 \times 20}{{\cdot}3} = 6000 \text{ lbs.}$$

Ex. 3.

Here proceeding as in the last example, we find $4500 \times {\cdot}3 = P \times 1{\cdot}5 \times 20$,

$$\therefore P = \frac{4500 \times {\cdot}3}{1{\cdot}5 \times 20} = 45 \text{ lbs.}$$

Ex. 4.

Let $x =$ the No. of teeth; then, supposing the axle to revolve once, we have

$$\text{Work applied} = 1 \times 2{\cdot}8 \times 3{\cdot}1416 \times x;$$

$$\text{Work done} = 280 \times {\cdot}4 \times 3{\cdot}1416;$$

$$\therefore\ 1 \times 2{\cdot}8 \times 3{\cdot}1416 \times x = 280 \times {\cdot}4 \times 3{\cdot}1416\,;$$
$$\therefore\ 2{\cdot}8 \times x = 280 \times {\cdot}4$$
$$\therefore\ x = \frac{280 \times {\cdot}4}{2{\cdot}8} = 40.$$

Hydrostatic Press.

ART. 47.

EX. 2.

By Ex. 1., a pressure of 1 ton is applied to the small piston; then the number of times which this pressure is increased by the fluid $= \frac{\text{area large piston}}{\text{area small piston}} = \frac{600}{\frac{3}{4}} = 800.$ But as 1 ton is applied to the small piston, the pressure produced upon the large piston will be 800 tons.

ACCUMULATED WORK.

Force of Gravity.

ART. 50.

Ex. 2.

Velocity acquired in 1 sec. $= 32\frac{1}{6}$ ft.

„ „ 3 sec. $= 3$ times $32\frac{1}{6}$ ft.

$= 96\frac{1}{2}$ ft.

Ex. 4.

$$\text{No. seconds} = \frac{70}{32\frac{1}{6}} = \frac{420}{193} = 2{\cdot}17.$$

Ex. 6.

$$\text{Space} = 3^2 \times 16\tfrac{1}{12} = 9 \times 16\tfrac{1}{12} = 144\tfrac{3}{4} \text{ feet.}$$

Ex. 7.

$$\text{Height of tower} = 4^2 \times 16\tfrac{1}{12} = 16 \times 16\tfrac{1}{12} = 257\tfrac{1}{3} \text{ feet.}$$

Ex. 9.

Space due to projection $= 2 \times 5 = 10$ ft.;

„ „ gravity $= 2^2 \times 16\frac{1}{12} = 64\frac{1}{3}$ ft.;

$\therefore$ Total space $= 10 + 64\frac{1}{3} = 74\frac{1}{3}$ ft.

Ex. 11.

Here, from the general formula, we have

$$S = t^2 \times 16\tfrac{1}{12},$$

$$\therefore 40 = t^2 \times 16\tfrac{1}{12},$$

$$\therefore\ t^2 = \frac{40}{16\frac{1}{12}} = \frac{480}{193} = 2{\cdot}48 + .$$

$$\therefore\ t = \sqrt{2{\cdot}48} = 1{\cdot}57 \text{ sec.}$$

Ex. 13.

$$\text{Time of the body's fall} = \frac{386}{32\frac{1}{6}} = 12 \text{ seconds.}$$

$$\therefore\ \text{Space} = 12^2 \times 16\tfrac{1}{12} = 144 \times 16\tfrac{1}{12} = 2316 \text{ ft.}$$

Projectiles.

ART. 51.

Ex. 2.

$$\text{Time to fall 80 ft.} = \sqrt{\frac{80}{16\frac{1}{12}}} = 2{\cdot}23 + \text{sec.};$$

$$\therefore\ \text{Distance} = \text{velo. projection} \times \text{time in falling}$$
$$= 90 \times 2{\cdot}23 = 200{\cdot}7 \text{ feet.}$$

Work accumulated in a Moving Body.

ART. 52.

Ex. 5.

$$\text{Work in the body, or U} = \frac{50^2 \times 10}{2 \times 32\frac{1}{6}} = \frac{25000}{64\frac{1}{3}} = \frac{75000}{193} = 388 + ;$$

Resistance of friction $= \frac{1}{10}$ of 10 lbs.;

$\therefore$ Work destroyed by friction in moving the body over 1 foot $= \frac{1}{10}$ of $10 \times 1 = 1$.

$$\therefore\ \text{No. ft.} = \frac{388}{1} = 388 \text{ ft.}$$

Or thus,

Let $x =$ the space in feet; then

Resistance friction $= \frac{1}{10}$ of 10 lbs. $= 1$ lb.

Work destroyed by friction $= 1 \times x = x$.

But, from the foregoing solution, the work accumulated in the body is 388; and this work must all be destroyed before the body can come to a state of rest.

$$\therefore\ x = 388 \text{ feet.}$$

Ex. 7.

$$\text{Speed per sec.} = \frac{40 \times 5280}{60 \times 60} = \frac{176}{3} \text{ ft.}$$

$$\therefore \text{ Work in the train, or } U = \frac{\left(\frac{176}{3}\right)^2 \times 60 \times 2240}{2 \times 32\frac{1}{6}} =$$

$$\frac{176 \times 176 \times 60 \times 2240}{3 \times 3 \times 2 \times 32\frac{1}{6}} = 7190283.$$

Work due to friction in moving the train over 1 foot $= 8 \times 60 \times 1 = 480$;

Work due to gravity in moving the train over 1 foot $= 60 \times 2240 \times \frac{1}{100} = 1344$;

Total work of the resistance over 1 foot $= 480 + 1344 = 1824$.

$$\therefore \text{ No. ft.} = \frac{7190283}{1824} = 3942 \text{ ft.} = \cdot 74 \text{ miles.}$$

Or thus,

Let $x =$ the distance in feet to which the train will ascend the incline.

Rise of the rail in 100 feet $= 1$ foot;

$$\therefore \quad ,, \qquad x \text{ feet} = \frac{x}{100} \text{ feet.}$$

Work due to friction $= 8 \times 60 \times x$;

Work due to gravity $= 60 \times 2240 \times \frac{x}{100}$;

$\therefore$ Total work of the resistances when the train stops

$$= 8 \times 60 \times x + 60 \times 2240 \times \frac{x}{100}$$
$$= x \times (8 \times 60 + 60 \times 22{\cdot}4)$$
$$= x \times 1824.$$

Now, by the foregoing solution, the work accumulated in the train is 7190283 ;

$$\therefore x \times 1824 = 7190283 ;$$
$$\therefore x = \frac{7190283}{1824} = 3942 \text{ ft.} = {\cdot}74 \text{ miles.}$$

Ex. 8.

$$\text{Speed per sec.} = \frac{20 \times 5280}{60 \times 60} = \frac{88}{3} \text{ ft.}$$
$$\therefore \text{Work in train, or } U = \frac{\left(\frac{88}{3}\right)^2 \times 60 \times 2240}{2 \times 32\frac{1}{6}} = 1797519.$$

Work due to friction in moving the train over 1 foot $= 8 \times 60 \times 1 = 480$.

Work due to gravity in moving the train over 1 foot $= 60 \times 2240 \times \frac{1}{30} = 4480$.

$\therefore$ Total work of the resistance over 1 foot $= 480 + 4480 = 4960$.

$$\therefore \text{No. ft.} = \frac{1797519}{4960} = 362{\cdot}4 \text{ ft.}$$

Or thus,

Let $x =$ the distance in feet to which the train will ascend the incline.

Rise of the rail in 30 feet = 1 foot;

$\therefore$ „ „ x feet $= \frac{x}{30}$ feet.

Work due to friction $= 8 \times 60 \times x = 480 \times x$;

„ „ gravity $= 60 \times 2240 \times \frac{x}{30} = 4480 \times x$;

$\therefore$ Total work of the resistances $= 480 \times x + 4480 \times x = 4960 \times x$.

Now, by the foregoing solution, the work to be destroyed is 1797519.

$$\therefore 4960 \times x = 1797519,$$

$$\therefore x = \frac{1797519}{4960} = 362{\cdot}4 \text{ ft.}$$

Ex. 11.

Work in the ball, or $U = \frac{200^2 \times 4}{2 \times 32\frac{1}{6}}$ (to clear of fractions)

$$= \frac{200^2 \times 4 \times 3}{6 \times 32\frac{1}{6}} = 2487.$$

Now the pressure of 1 lb. upon the ball would produce 10 units of work.

$$\therefore \text{Mean pressure on the ball} = \frac{2487}{10} = 248{\cdot}7 \text{ lbs.}$$

Or thus,

Let $x =$ the mean pressure on the ball in lbs.
Work of the elastic vapour $= x \times 10$.
But this is equal to the work accumulated in the ball.

$$\therefore x \times 10 = 2487;$$

$$\therefore x = \frac{2487}{10} = 248{\cdot}7 \text{ lbs.}$$

ADDITIONAL EXERCISES.

Ex. 15.

A carriage of 1 ton moves on a level rail with the speed of 9 ft. per second; what will be its velocity after it has moved over the space of 300 feet, supposing friction to be 6 lbs. per ton?

$$\text{Work accumulated in the carriage} = \frac{9^2 \times 2240}{64\frac{1}{3}}$$

$$\text{Work lost by friction} = 6 \times 300 = 1800.$$

$$\therefore \text{Work remaining in the carriage} = \frac{9^2 \times 2240}{64\frac{1}{3}} - 1800.$$

Let $v =$ the velocity of the carriage in feet per second; then

Work accumulated remaining in the carriage $= \frac{v^2 \times 2240}{64\frac{1}{3}}$,

$$\therefore \frac{v^2 \times 2240}{64\frac{1}{3}} = \frac{9^2 \times 2240}{64\frac{1}{3}} - 1800,$$

$$\therefore v = \sqrt{\frac{9^2 \times 2240 - 1800 \times 64\frac{1}{3}}{2240}} = 5{\cdot}4 + \text{ ft.}$$

Ex. 16.

What would be the velocity of the carriage in Ex. 15. after moving over the space of 400 feet?

Here we have

$$\frac{v^2 \times 2240}{64\frac{1}{3}} = \frac{9^2 \times 2240}{64\frac{1}{3}} - 2400\,;$$

$$\therefore v = \sqrt{\frac{9^2 \times 2240 - 2400 \times 64\frac{1}{3}}{2240}}$$

$$= \sqrt{12{\cdot}07} = 3{\cdot}47 \text{ ft.}$$

Work in a Rotating Body.—Centre of Gyration.

Art. 53.

Ex. 2.

Space described by the wt. in 1 revo. $= 5 \times 2 \times 3{\cdot}1416 = 31{\cdot}416$ ft.

$\therefore$ Velocity of the weight per sec. $= \frac{31{\cdot}416 \times 10}{60} = 5{\cdot}236$ ft.

$\therefore$ Work in the wheel $= \frac{5{\cdot}236^2 \times 10 \times 112}{2 \times 32\frac{1}{6}} = 477{\cdot}28$.

Circumference axis $= \frac{3}{12} \times 3{\cdot}1416 = {\cdot}7854$.

$\therefore$ Work destroyed by friction in 1 revo. $=$ friction $\times$ circum. axis $= \frac{1120}{7} \times {\cdot}7854 = 125{\cdot}664$.

$$\therefore \text{No. revolutions} = \frac{477{\cdot}28}{125{\cdot}664} = 3{\cdot}8.$$

Ex. 4.

By Ex. 3., the work in the wheel $= 1954{\cdot}9$; and the work in 1 lift of the hammer, exclusive of friction, $= 400$. Now we have to find the work destroyed by friction in 1 revolution of the wheel in order to determine the total work of the resistance.

Pressure wheel upon the axis $= \frac{1}{8}$ of $4 \times 112 = 56$ lbs.

Work destroyed by friction in 1 revo. $= 56 \times \frac{6}{12} = 28$.

As the hammer makes one lift in every revolution of the wheel, we have

Work of the resistances in 1 revo. = work of the hammer + work of friction $= 400 + 28 = 428$.

$$\therefore \text{No. lifts} = \frac{\text{work in wheel}}{\text{work of resistances in 1 revo.}}$$

$$= \frac{1954{\cdot}9}{428} = 4{\cdot}5 +.$$

Ex. 6.

The whole weight of the stone is collected in a point which is called the centre of gyration, but we have

Dist. centre of gyration from the axis = radius $\times \sqrt{\frac{1}{2}}$ $= \frac{3}{2} \times \sqrt{\frac{1}{2}}$ ft.

Velocity of a point 1 ft. from the axis $= 2 \times 3{\cdot}1416 = 6{\cdot}2832$ ft.

$\therefore$ Velocity wt., or a point $\frac{3}{2}\sqrt{\frac{1}{2}}$ ft. from the axis $= 6{\cdot}2832 \times \frac{3}{2}\sqrt{\frac{1}{2}}$ ft. $= 9{\cdot}4248 \times \sqrt{\frac{1}{2}}$.

The square of this velocity $= 9{\cdot}424^2 \times \frac{1}{2} = 44{\cdot}405$.

$$\therefore \text{Work in the stone} = \frac{44{\cdot}405 \times 300}{2 \times 32\frac{1}{6}} = 207{\cdot}05.$$

Circum. axis $= \frac{1}{12} \times 3{\cdot}1416$ ft

Pressure on the axis $= \frac{1}{8}$ of 300 lbs.

$\therefore$ Work destroyed in 1 revo. by friction $= \frac{1}{12} \times 3{\cdot}1416 \times \frac{1}{8} \times 300 = 9{\cdot}81$.

$$\therefore \text{No. revo.} = \frac{207{\cdot}05}{9{\cdot}81} = 21{\cdot}09.$$

Maximum Velocity of the Piston of an Engine.

ART. 56.

Ex. 2.

Let $x =$ the point of the stroke, then as the velocity of the piston will be greatest when the pressure of the steam is 20 lbs., we have

$$x \times 20 = 2 \times 40,$$

$$\therefore x = \frac{2 \times 40}{20} = 4 \text{ ft.}$$

Ex. 5.

By Ex. 4., Art. **25.**, the mean pressure of the steam = 14·9 lbs., and therefore this must also be the pressure of the steam at that point in the stroke where the velocity of the piston is a maximum. Putting $x =$ the distance of this point from the bottom of the cylinder, we have

$$x \times 14{\cdot}9 = 1 \times 30,$$

$$\therefore x = 2 \text{ feet nearly.}$$

The steam will therefore act expansively over 1 foot of the stroke. Dividing this space into 4 equal parts, the interval between each will be $\frac{1}{4}$ ft.; hence we have after the method of Ex. 2., page 37.

$$P = 30 \text{ lbs.}$$

$$P_1 = \frac{30 \times 1}{1\frac{1}{4}} = \frac{120}{5} = 24 \text{ lbs.}$$

$$P_2 = \frac{30 \times 1}{1\frac{1}{2}} = \frac{60}{3} = 20 \text{ lbs.}$$

$$P_3 = \frac{30 \times 1}{1\frac{3}{4}} = \frac{120}{7} = 17{\cdot}14 \text{ lbs.}$$

$$P_4 = \frac{30 \times 1}{2} = 15 \text{ lbs.}$$

$\therefore$ Work done expansively on 1 inch of the piston up to the point of maximum velocity

$$= \frac{\frac{1}{4}}{3}\{30 + 15 + 4(24 + 17{\cdot}14) + 2 \times 20\}$$

$$= \frac{1}{12} \times 249{\cdot}56 = 20{\cdot}8 \text{ nearly.}$$

Space moved over by the piston before the steam is cut off $= 1 - \frac{1}{4} = \frac{3}{4}$ ft.

$\therefore$ Work done on 1 inch of the piston before the steam is cut off $= 30 \times \frac{3}{4} = 22{\cdot}5$.

$\therefore$ Total work on 1 inch of the piston up to the point of maximum velocity $= 20{\cdot}8 + 22{\cdot}5 = 43{\cdot}3$.

$\therefore$ Work on the whole piston up to this point $= 43{\cdot}3 \times 1440 = 62352$.

We have now to find the work absorbed by the resistances.

Length of the stroke $= 2 - \frac{1}{4} = 1\frac{3}{4}$ ft.

$\therefore$ Work done on the resistances = mean pressure steam × length of stroke × area piston $= 14{\cdot}9 \times 1\frac{3}{4} \times 1440 = 37548$.

$\therefore$ Work remaining in the piston when it arrives at the point of max. velocity = work steam − work resistance $= 62352 - 37548 = 24804$, which is the work accumulated in the engine.

$$\therefore \frac{V^2 \times 30000}{64\frac{1}{3}} = 24804;$$

$$\therefore V = \sqrt{\frac{24804 \times 64\frac{1}{3}}{30000}} = 7{\cdot}2 \text{ ft.}$$

Velocity acquired by a Body descending an inclined Plane.

ART. 57.

Ex. 2.

No. seconds to descend 8 feet $= \sqrt{\frac{8}{16\frac{1}{12}}}$,

See Ex. 10. Art. 50. of the text;

$$\therefore \text{Velocity acquired} = \sqrt{\frac{8}{16\frac{1}{12}}} \times 32\frac{1}{6} = 22{\cdot}6 \text{ ft.}$$

Work of the Fire Engine.

ART. 62.

EX. 3.

Weight of water discharged per sec. $= \frac{1}{144} \times 120 \times 62{\cdot}5$

" " " per min. $= \frac{1}{144} \times 120 \times 62{\cdot}5 \times 60$

$= 3125$ lbs.

$$\therefore \text{Accumulated work per min.} = \frac{120^2 \times 3125}{2 \times 32\frac{1}{6}} = \frac{45000000}{64\frac{1}{3}}$$

$$= 699481.$$

Poncelet's Water Wheel.

ART. 63.

EX. 2.

Weight water flowing per sec. $= 70 \times 62{\cdot}5 = 4375$ lbs.

$$\therefore \text{Work per sec.} = \frac{6^2 \times 4375}{64\frac{1}{3}} = 2448{\cdot}18.$$

$$\therefore \text{Work per min.} = 2448{\cdot}18 \times 60 = 146890.$$

$$\therefore \text{H. P.} = \frac{146890}{33000} = 4{\cdot}4 + .$$

EX. 4.

Here the velocity with which the water leaves the paddles $= 9 - 2 \times 4 = 1$ foot.

$$\therefore \text{Work remaining in the water} = \frac{1^2 \times 60 \times 62{\cdot}5}{64\frac{1}{3}}.$$

$$\text{Work in the water at first} = \frac{9^2 \times 60 \times 62{\cdot}5}{64\frac{1}{3}}.$$

∴ Work done on the wheel per sec.

$$= \frac{9^2 \times 60 \times 62{\cdot}5}{64\frac{1}{3}} - \frac{1^2 \times 60 \times 62{\cdot}5}{64\frac{1}{3}} = \frac{80 \times 60 \times 62{\cdot}5}{64\frac{1}{3}} = 4663.$$

$$\therefore \text{H. P.} = \frac{4663 \times 60}{33000} = 8{\cdot}47.$$

Or thus, by the formula, $U = \frac{2W}{g}(V - v)v$.

Where U is the work per sec.; $W = 60 \times 62{\cdot}5$; $V = 9$; $v = 4$; and $g = 32\frac{1}{6}$;

$$\therefore U = \frac{2 \times 60 \times 62{\cdot}5}{32\frac{1}{6}} \times (9-4) \times 4 = \frac{2 \times 60 \times 62{\cdot}5 \times 5 \times 4}{32\frac{1}{6}}$$

$=4663$, which is the work per sec.; and so on as before.

THE FLY WHEEL.

ART. 65.

Ex. 2.

Here, by the general formula,

$$W = \frac{n\,U}{k^2\,N^3} \times 808{\cdot}2 \text{ lbs.},$$

$$W = \frac{15 \times 330000}{10^2 \times 20^3} \times 808{\cdot}2 \text{ lbs.} = 2{\cdot}2 \text{ tons.}$$

EQUILIBRIUM OF PRESSURES, &c.

ART. 67.

Ex. 1.

Construct the right-angled triangle ABD (see fig. p. 66. of the text), having $AB = 3$, $BD = 5$; take $BC = 2$; draw CO perpendicular to AB, intersecting AD in O; take $On = 1\frac{1}{2}$; join OB; and complete the parallelogram *nreO*. Then the units in $Or = \cdot 8$ tons, which is the pressure on the hinge; $Oe = 1{\cdot}16$ tons, which is the tension of the chain.

Ex. 2.

The construction of the triangle ABD, &c., being the same as in the last case, take $On = 8$, from any convenient scale, and complete the parallelogram of pressures *nreO* as before; then $Or = 4{\cdot}3$ cwts., and $Oe = 6{\cdot}2$ cwts.

Ex. 4.

Here O*e* is taken 32 units from any convenient scale; and so on as in Ex. 3.; then the units in O*n*, measured from the same scale, are 41, which is the number of cwts. acting on the chain.

ART. 68.

Ex. 2.

Here having constructed the triangle AOD, &c. (see fig. p. 65. of the text), D*n* is taken 15 units, and the parallelogram D*n*I*e* is constructed; then the units in D*e*

$= 16$, which is the number of cwts. acting on the chain; and D I $= 29$, which is the number of cwts. acting on the pole.

Ex. 4.

Here D*e* is taken equal to 22, and the parallelogram *n* I *e* D is constructed; then the units in D *n* are found to be 20 nearly, which is the number of cwts. in the weight W.

Art. 70.

Ex. 1.

Construct the rectangle A O H, having O H $= 5$, and O A $= 4$; take A D $= \frac{1}{2}$ of $5 = 2\frac{1}{2}$; join D O, and draw D C parallel to A O; take D C $= 2$ units, from any convenient scale, and complete the parallelogram D C Q F; then D Q $= 2{\cdot}35$ units, which is the number of cwts. pressure on the pin O, and D F $= 1{\cdot}25$ units, which is the number of cwts. pressure on A.

Ex. 2.

Here D C is taken 160 units, and so on as in the last case.

To determine whether a pillar will stand or fall when acted upon by a given pressure.

Art. 71.

Ex. 2.

$$\text{Weight pillar} = 20 \times 2 \times 1 \times 120 = 4800.$$

Having constructed the rectangle, representing the section of the pillar, on as large a scale as convenient, from the same scale take R P $= 1{\cdot}5$ units, and draw P A C, making the angle C P R $= 30°$. Considering the units on the scale to be thousands, take A B $= 4{\cdot}8$, A C $= {\cdot}4$, and complete the parallelogram A B D C. Join A D, and produce it until it intersects the base V O produced. Now as this point falls without the base, that is, to the left of O, the pillar will fall.

Ex. 4.

In this case the angle C P R is taken 45°, the other parts of the construction being the same as in Ex. 3., then the units in A C will be found to be about 303, which is the number of lbs. pressure in the thrust P.

Pressure of Roofs.

ART. 73.

Ex. 2.

Construct the isosceles triangle A C Q (see *fig.* p. 112. of the text) having the base C Q = 32, and the angle, A C Q, at the base equal to 45°; then the length of the rafter A C or A Q = 22·6; and the weight of 1 ft. of the roof = 22·6 × 2 × 12 = 542 lbs. + ; hence W = $\frac{1}{2}$ of 542 lbs. = 271 lbs. From any convenient scale take A O = 271, and construct the parallelogram A *e* O *n*; then A *e* or A *n*, measured off from the same scale, will be found to contain about 192 units, which is the number of lbs. thrust upon the rafter.

ART. 74.

Ex. 1.

Take C *a* = 148 units; draw *a* P perpendicular to C Q, and complete the parallelogram *a t* C P; then C P = 256, which is the units of lbs. tension of the tie-beam. Again take C *a* = 192; and so on as in the preceding case; then C P = 271, which is the units of lbs. tension of the tie-beam.

ART. 75.

Ex. 2.

Proceeding in the same manner as in Ex. 1. Art. 73., we find the length of the rafter = 56·5 ft.; the weight of each rafter 680 lbs. nearly; and the thrust upon the rafter = 480 lbs. Now take C a = 480, C t = $\frac{1}{2}$ of 680 = 340, and construct the parallelogram $C\,t\,P_1\,a$; then the units in $C P_1 = 760$ nearly, which is the lbs. pressure of the roof, acting in the direction P_1 C, and tending to overturn the side wall.

Ex. 3.

In this case, the length of the rafter = 14·1 ft.; the weight of each rafter = 170 lbs. nearly; and the thrust upon the rafter = 120 lbs. Constructing the parallelogram $C\,t\,P_1\,a$ as before, we find, $C\,P_1$ = 190 lbs.

Ex. 5.

Weight of 1 ft. length of the wall = $1 \times 1 \times 15 \times 120$ = 1800 lbs. Having constructed the figure as in Ex. 4., with this difference, that A C (see *fig.* page 111. of the text) is taken 1800 in the place of 3600, we find that the resultant pressure A D cuts the base of the wall to the left of the edge O: in this case therefore the wall would fall.

Pressure of Fluids on Embankments.

Art. 76.

Ex. 2.

Area surface upon which the water presses = $9 \times 1 = 9$ sq. ft.

∴ Pressure water = $9 \times \frac{1}{2}$ of $9 \times 62·5 = 2531\frac{1}{4}$ lbs.

Ex. 4.

Circumference of the barrel = $3 \times 3·1416 = 9·4248$ ft.;

∴ Surface of the staves = $9·4248 \times 4 = 37·6992$ sq. ft.

∴ Pressure water = $37·699 \times \frac{1}{2}$ of $4 \times 62·5 = 4712$ lbs. nearly.

ART. 77.

Ex. 1.

Distance centre of pressure from the bottom $= \frac{1}{3}$ of 6 ft. $= 2$ ft.

Ex. 2.

Distance centre of pressure from the bottom $= \frac{1}{3}$ of 9 ft. $= 3$ ft.

ART. 78.

Ex. 2.

Surface upon which the water presses $= 12 \times 1 = 12$ sq. ft.;

Pressure of the water $= 12 \times \frac{1}{2}$ of $12 \times 62{\cdot}5 = 4500$ lbs.;

Distance centre of pressure from the bottom, or $OI = \frac{1}{3}$ of 12 ft. $= 4$ft. (See *fig.* p. 116. of the text.)

$\therefore$ Moment of the pressure of the water $=$ pressure water $\times OI = 4500 \times 4 = 18000$.

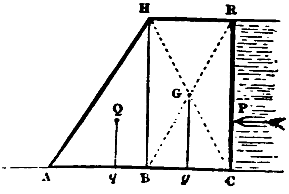
Weight of the embankment $= 12 \times 5 \times 1 \times 120 = 7200$ lbs.;

$\therefore$ Moment of the wt. of the embankment $=$ wt. $\times OV = 7200 \times \frac{1}{2}$ of $5 = 18000$.

Here we find that the moment of the pressure of the water is exactly the same as the moment of the weight of the embankment; hence it will be just upon the point of overturning.

Ex. 6.

Let the embankment, A C R H, be divided into two portions by the line H B drawn parallel to R C; let Qq be a vertical line passing through Q the centre of gravity of the triangular portion A B H, and Gg a vertical line passing through G the centre of gravity of the *rectangular* portion B C R H; then we have



$$Aq = \tfrac{2}{3} \text{ of } AB = \tfrac{2}{3} \text{ of } 6 = 4 \text{ ft.};$$

$$Ag = AC - gC = AC - \tfrac{1}{2}\,BC = 10 - \tfrac{1}{2} \text{ of } 4 = 8 \text{ ft.};$$

$$\text{Volume } ABH = \tfrac{1}{2}\,AB \times BH \times 1 = 3 \times 15 \times 1 = 45 \text{ c. ft.}$$

$$\therefore \text{ Wt. } ABH = 45 \times 100 = 4500 \text{ lbs.}$$

$$\text{Volume } BHRC = BC \times BH \times 1 = 4 \times 15 \times 1 = 60 \text{ c. ft};$$

$$\therefore \text{ Wt. } BHRC = 60 \times 100 = 6000 \text{ lbs.}$$

$$\text{Moment } ABH = \text{wt.} \times Aq = 4500 \times 4 = 18000.$$

$$\text{Moment } BHRC = \text{wt.} \times Ag = 6000 \times 8 = 48000.$$

$$\therefore \text{ Moment of the whole embankment} = 18000 + 48000 = 66000.$$

Pressure of the water on the embankment

$$= 15 \times 1 \times \tfrac{1}{2} \text{ of } 15 \times 62{\cdot}5 = 7031{\cdot}25 \text{ lbs.}$$

$\therefore$ Moment of the pressure of the water

$$= \text{pressure} \times CP = 7031{\cdot}25 \times \tfrac{1}{3} \text{ of } 15 = 35156{\cdot}25.$$

Here we find that the moment of the embankment is greater than that of the water, — hence the structure will stand.

Ex. 9.

Pressure of the water on each half of the gate

$$= 4 \times 7 \times \tfrac{1}{2} \text{ of } 7 \times 62{\cdot}5 = 6125 \text{ lbs.}$$

Taking the figure described in Ex. 8., p. 119. of the text, we have,

$$DP = \tfrac{1}{3} \text{ of } 7 \text{ ft.} = \tfrac{7}{3} \text{ ft.};$$

$$DB \text{ or } AC = \tfrac{1}{2} \text{ of } (CD - AB) = \tfrac{1}{2} \text{ of } (7 - 4) = \tfrac{3}{2} \text{ ft.};$$

$$\therefore\ BP = DP - DB = \tfrac{7}{3} - \tfrac{3}{2} = \tfrac{5}{6} \text{ ft.}$$

$$\therefore\ AP = AB - BP = 4 - \tfrac{5}{6} = 3\tfrac{1}{6} \text{ ft.}$$

Now we have on the principle of the lever,

$$\text{Pressure on } B \times BA = \text{pressure on } P \times PA,$$

$$\therefore \text{ Pressure on } B \times 4 = 6125 \times 3\tfrac{1}{6};$$

$$\text{Pressure on } B = \frac{6125 \times 3\frac{1}{6}}{4} = 4849 \text{ lbs.}$$

ART. 79.

Ex. 2. (*Fig.* p. 116. of the text.)

Volume of the embankment $= 2 \times 1 \times 12 = 24$ c. ft.;

$\therefore$ Wt. „ „ $= 24 \times 130 = 3120$ lbs.

Moment of embankment $=$ wt. $\times$ OV $= 3120 \times \frac{1}{2}$ of 2 $= 3120$.

Moment of the thrust on the stay $=$ thrust $\times$ OT $=$ thrust $\times 2{\cdot}4$.

Pressure of the water $= 12 \times 1 \times \frac{1}{2}$ of $12 \times 62{\cdot}5$ $= 4500$ lbs.;

$\therefore$ Moment of the pressure of the water $=$ wt. $\times$ DP $= 4500 \times \frac{1}{3}$ of $12 = 18000$.

Hence we have by the equality of moments,

$$\text{Thrust} \times 2{\cdot}4 + 3120 = 18000,$$

$$\therefore \text{Thrust} = \frac{18000 - 3120}{2{\cdot}4} = 6200 \text{ lbs.}$$

Ex. 3.

To find the perpendicular, OT, on the stay, we have given the length of the stay SY $= 10$ ft., and the distance, OS, of its foot from the edge of the embankment $= 8$ ft.; hence

$$OY = \sqrt{SY^2 - OS^2} = \sqrt{10^2 - 8^2} = 6 \text{ ft.}$$

Area triangle SOY $= \frac{1}{2}$ SO $\times$ OY $= \frac{1}{2}$ of $8 \times 6 = 24$; but we have also,

Area triangle SOY $= \frac{1}{2}$ SY $\times$ OT $= 5 \times$ OT;

$$\therefore 5 \times OT = 24,$$

$$\therefore OT = \frac{24}{5} = 4{\cdot}8 \text{ ft.}$$

The remaining portion of the data of the question being the same as in Ex. 1., we have

$$\text{Thrust} \times 4{\cdot}8 + 1755 = 18000,$$

$$\therefore \text{Thrust} = 3384 \text{ lbs.}$$

To determine the form of maximum, or equable, strength of an embankment.

ART. 80.

Ex. 2.

Let $x=$ the base QU, then

$$\text{Wt. QUHV} = x \times 1 \times 2 \times 100 = 200x;$$

$$\text{Moment QUHV} = 200x \times \frac{x}{2} = 100x^2.$$

Pressure water on $\text{HU} = 2 \times 1 \times \frac{1}{2}$ of $2 \times 62{\cdot}5 = 125$ lbs.;
Moment of the pressure of the water on HU

$$= \text{wt.} \times \tfrac{1}{3}\text{HU} = 125 \times \tfrac{1}{3} \text{ of } 2 = \frac{250}{3}.$$

But as QUHV is upon the point of being overturned on the edge Q by the pressure of the water, we have by the equality of moments,

$$100x^2 = \frac{250}{3},$$

$$\therefore\ x = \sqrt{\frac{250}{300}} = \sqrt{{\cdot}8333} = {\cdot}912 \text{ ft. nearly.}$$

Let $y = \text{FD}$; then we have

$$\text{DQ} = \text{DV} - \text{QV} = 6 - 2 = 4 \text{ ft.}$$

$$\text{Wt. FDQ} = \text{FD} \times 1 \times \text{DQ} \times 100$$
$$= y \times 1 \times 4 \times 100 = 400y;$$

$$\therefore \text{ Moment FDQ} = \text{wt.} \times \tfrac{1}{2}\text{FD} = 400y \times \tfrac{1}{2}y$$
$$= 200y^2.$$

$$\text{Wt. DNHV} = \text{QU} \times 1 \times \text{NH} \times 100$$
$$= {\cdot}912 \times 6 \times 100 = 547{\cdot}2 \text{ lbs.}$$

$$\therefore \text{ Moment DNHV} = \text{wt.} \times \{\text{FD} + \tfrac{1}{2}\text{DN}\}$$
$$= 547{\cdot}2 \times \{y + \tfrac{1}{2} \text{ of } {\cdot}912\}$$
$$= 547{\cdot}2y + 547{\cdot}2 \times {\cdot}456.$$

$\therefore$ Moment of the whole embankment $=$ moment FDQ $+$ moment $\text{DNHV} = 200y^2 + 547{\cdot}2y + 547{\cdot}2 \times {\cdot}456.$

$$\text{Pressure water on } HN = 6 \times 1 \times \tfrac{1}{2} \text{ of } 6 \times 62{\cdot}5$$
$$= 1125 \text{ lbs.};$$

$\therefore$ Moment water in overturning $HNFV$ = pressure $\times \frac{1}{3}$ HN
$$= 1125 \times \tfrac{1}{3} \text{ of } 6 = 2250.$$

$$\therefore\ 200y^2 + 547{\cdot}2y + 547{\cdot}2 \times {\cdot}456 = 2250.$$

Solving this quadratic equation we find $y = 2{\cdot}088$, and $FN = 2{\cdot}088 + {\cdot}912 = 3$ ft.

Revetment Walls.

Art. 81.

Ex. 2.

Wt. of the equivalent fluid $= 100 \times {\cdot}1716 = 17{\cdot}16$ lbs.

$\therefore$ Pressure of the earth upon 1 foot length of wall

$$= 40 \times 1 \times \frac{40}{2} \times 17{\cdot}16 = 13728.$$

Moment of the pressure of the earth in overturning the wall = pressure $\times \frac{1}{3}$ height $= 13728 \times \frac{1}{3}$ of $40 = 183040$.

Wt. of the wall $= 40 \times 8 \times 1 \times 120 = 38400$ lbs.;

$\therefore$ Moment of the wt. of the wall = wt. $\times \frac{1}{2}$ thickness
$$= 38400 \times \tfrac{1}{2} \text{ of } 8 = 153600.$$

Here the moment of the pressure of the earth is greater than that of the wall; hence the wall would fall.

Ex. 7.

(1.) To find the moment of the wall.

Weight of the wall $= 24 \times 1 \times 6 \times 130 = 18720$.

Moment of the wall = wt. $\times \frac{1}{2}$ of thickness
$$= 18720 \times \tfrac{1}{2} \text{ of } 6 = 56160.$$

(2.) To find the moment of the sloping earth.

Wt. of a c. ft. of the equivalent fluid, with respect to the sloping earth $= 120 \times \frac{1}{2} = 60$ lbs.;

Pressure of this earth $= 5 \times 1 \times \frac{5}{2} \times 60 = 750$ lbs.

$$\text{Moment of this pressure} = \text{pressure} \times \tfrac{1}{3} \text{ of height}$$
$$= 750 \times \tfrac{1}{3} \text{ of } 5 = 1250.$$

$\therefore$ Sum of the moments sustaining the wall = moment wall + moment sloping earth = 56160 + 1250 = 57410.

(3.) To find the moment of the level earth.

Wt. of a c. ft. of the equivalent fluid = 120 × ·1716;

$$\text{Pressure of the earth} = 24 \times 1 \times \tfrac{1}{2} \text{ of } 24 \times 120 \times \cdot 1716$$
$$= 5930{\cdot}496 \text{ lbs.}$$

$$\text{Moment of the earth} = \text{pressure} \times \tfrac{1}{3} \text{ height}$$
$$= 5930{\cdot}496 \times \tfrac{1}{3} \text{ of } 24 = 47443{\cdot}968.$$

As this moment is less than the sum of the moments sustaining the wall, we conclude that the structure will stand.

Ex. 8.

Let x = the thickness of the wall in feet; then, as in Ex. 6., we have

$$\text{Moment sloping earth} = 1250,$$
$$\text{and Moment level earth} = 92664.$$
$$\text{Wt. wall} = 30 \times 1 \times x \times 130 = 3900x;$$
$$\therefore \text{ Moment wall} = 3900x \times \frac{x}{2} = 1950x^2.$$

Now when the wall is upon the point of being overturned, the sum of the moments of the wall and the sloping earth is equal to the moment of the level earth;

$$\therefore\ 1950x^2 + 1250 = 92664,$$
$$\therefore\ x = \sqrt{\frac{92664 - 1250}{1950}}$$
$$= \sqrt{46{\cdot}8} = 6{\cdot}8 + \text{feet},$$

which is the thickness required.

Ex. 9.

Let x = the height of the sloping earth in feet; then, as in Ex. 6., we have

$$\text{Moment of the wall} = 70200,$$
$$\text{and Moment of the level earth} = 92664.$$

$$\text{Pressure of the sloping earth} = x \times 1 \times \frac{x}{2} \times 60 = 30x^2;$$

$$\therefore \text{ Moment of this earth} = \text{pressure} \times \tfrac{1}{3} \text{ height}$$

$$= 30x^2 \times \frac{x}{3} = 10x^3.$$

Now, we have

Moment wall + moment sloping earth = moment level earth,

$$\therefore\ 70200 + 10x^3 = 92664,$$

$$\therefore\ x^3 = \frac{22464}{10} = 2246{\cdot}4,$$

$$\therefore\ x = \sqrt[3]{2246{\cdot}4} = 13 \text{ ft.} +$$

Floating Bodies.—Specific Gravity.

ART. 82.

Ex. 2.

Here we have for the interior dimension of the barge,

$$\text{Length} = 10 \text{ ft.} - 2 \times 3 \text{ in.} = 10 \text{ ft.} - 6 \text{ in.} = 9\tfrac{1}{2} \text{ ft.},$$

$$\text{Breadth} = 5 \text{ ft.} - 2 \times 3 \text{ in.} = 4\tfrac{1}{2} \text{ ft.},$$

$$\text{Depth} = 4 \text{ ft.} - 3 \text{ in.} = 4 \text{ ft.} - \tfrac{1}{4} \text{ ft.} = 3\tfrac{3}{4} \text{ ft.},$$

$$\text{Content exterior solid} = 10 \times 5 \times 4 = 200 \text{ c. ft.}$$

$$\text{,, interior ,,} = 9\tfrac{1}{2} \times 4\tfrac{1}{2} \times 3\tfrac{3}{4}$$

$$= \frac{19 \times 9 \times 15}{2 \times 2 \times 4} = 160{\cdot}31 \text{ c. ft.}$$

$$\therefore \text{ Timber in the barge} = 200 - 160{\cdot}31 = 39{\cdot}69 \text{ c. ft.}$$

$$\therefore \text{ Wt. of timber in the barge} = \text{no. c. ft.} \times \text{wt. each c. ft.}$$

$$= 39{\cdot}69 \times 50 = 1984{\cdot}5 \text{ lbs.}$$

$$\text{Wt. of the displaced water} = 10 \times 5 \times \text{depth} \times 62{\cdot}5$$

$$= 3125 \times \text{depth}.$$

But since the weight of the displaced fluid is equal to the whole weight of the floating body, we have

$$3125 \times \text{depth} = 1984{\cdot}5 + 4 \times 2240;$$

$$\therefore \text{ depth} = \frac{1984{\cdot}5 + 4 \times 2240}{3125} = 3{\cdot}5 \text{ ft.}$$

Ex. 3.

Let $x=$ the load in tons; then we have by the final equation in Ex. 1., observing that the depth is 4 ft., and the load is to be found,

$$3125 \times 4 = 1353 + x \times 2240,$$

$$\therefore\ x = \frac{3125 \times 4 - 1353}{2240} = 4{\cdot}9 \text{ tons.}$$

Ex. 4.

In this case, we have, putting $x=$ the load in tons,

$$\text{Wt. displaced water} = 10 \times 5 \times 3 \times 62{\cdot}5 = 9375 \text{ lbs.};$$

$$\text{Load in lbs.} = x \times 2240;$$

And as in Ex. 1.,

$$\text{Wt. timber in the barge} = 1353 \text{ lbs.}$$

$$\therefore\ x \times 2240 + 1353 = 9375,$$

$$\therefore\ x = \frac{9375 - 1353}{2240} = 3{\cdot}58 \text{ tons.}$$

Ex. 5.

Here the weight of the displaced water must be equal to the weight of the barge.

Let $x=$ the depth to which it sinks in feet; then

$$\text{Wt. displaced water} = 10 \times 5 \times x \times 62{\cdot}5 = 3125x;$$

but the weight of the barge is 1353 lbs., as in Ex. 1.;

$$\therefore\ 3125x = 1353,$$

$$\therefore\ x = \frac{1353}{3125} = {\cdot}433 \text{ ft.}$$

Ex. 7.

Wt. lost in water $= 36 - 28 = 8$ gr.

But this is the weight of water equal in bulk to the body.

$\therefore$ No. times the body is heavier than water $= \frac{36}{8} = 4\frac{1}{2}$.

Wt. c. ft. water $= 1000$ oz.

$\therefore$ „ „ body $= 4\frac{1}{2}$ times 1000 oz. $= 4500$.

Ex. 10.

Here a cubic foot of brickwork weighs 2000 oz.;

$$\therefore \text{ No. c. ft.} = \frac{2 \times 2240 \times 16}{2000} = 35\cdot84.$$

Ex. 11.

$$\text{Wt. water in ounces} = 375 \times 16 = 6000 \text{ oz.};$$
$$\text{Wt. 1 c. ft. of water} = 1000 \text{ oz.};$$
$$\text{No. c. ft.} = \frac{6000}{1000} = 6 \text{ c. ft.}$$

Limiting angle of resistance.

Art. 83.

Ex. 1.

Here, the tangent of the angle $= \cdot7$, therefore, from a table of tangents, we find the angle to be 35°.

Ex. 2.

Having constructed the figure, as in Ex. 5., Art. 68., measure off the angle which V O makes with the vertical; this angle being less than 36° shows that the ladder will not slip.

Equilibrium of the lever, and the wheel and axle, taking the friction upon the axis into account.

Art. 84.

Ex. 2.

$$\begin{aligned}\text{Here } OI &= \sin OTI \times OT \\ &= \sin 40^\circ \times 3 \\ &= \cdot64278 \times 3 = 1\cdot92834\end{aligned}$$

$$PI = PO - IO = 14 - 1{\cdot}92834 = 12{\cdot}07166.$$
$$QI = QO + IO = 12 + 1{\cdot}92834 = 13{\cdot}92834.$$

Now by the equality of moments,

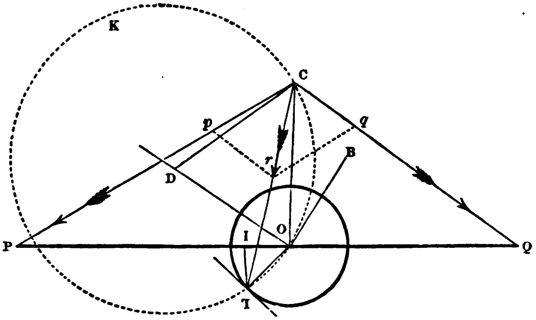
$$P \times PI = Q \times QI,$$
$$\therefore\ 1 \times 12{\cdot}07166 = Q \times 13{\cdot}92834,$$
$$\therefore\ Q = \frac{12{\cdot}07166}{13{\cdot}92834} = {\cdot}86 + \text{lbs.}$$

Ex. 3.

$$OI = \sin 35^\circ \times OT$$
$$= {\cdot}57357 \times 2 = 1{\cdot}147.$$
$$PI = PO - IO = 15 - 1{\cdot}147 = 13{\cdot}853.$$
$$QI = QO + IO = 8 + 1{\cdot}147 = 9{\cdot}147.$$
$$\therefore\ P \times PI = Q \times QI,$$
$$\therefore\ 2 \times 13{\cdot}853 = Q \times 9{\cdot}147,$$
$$\therefore\ Q = \frac{2 \times 13{\cdot}853}{9{\cdot}147} = 3 \text{ lbs.} +.$$

Ex. 4.

Let PQ represent the lever; OT the radius of the axis;



PC and QC the directions of the pressures P and Q respectively; CT the resultant of these pressures cutting the axis at T, and forming, with OT, the angle of friction OTC =35°.

From a scale of equal parts take PO=14; OQ=12; with the radius OT=3, describe the circle representing the axis; draw PC making the angle QPC=30°, and QC making the angle PQC=35°; join CO.

Now upon CO as a chord we have to describe a circle which shall contain the angle OTC=35°, or the given angle of friction. Draw OB making the angle COB=35°, the angle of friction; draw OD perpendicular to OB; from C draw CD making the angle OCD equal to the angle COD; then these two lines will intersect in a point D, which is the centre of the circle required. (See the Author's Geometry and Mensuration, p. 55., Art. 60., Cor. 3.)

With D as a centre and radius equal to DC or DO, describe the circle KCOT intersecting the axis of the lever in the point T; join CT; then $\angle$ OTC=$\angle$COB=35°, the angle of friction; and CT will be the direction of the resultant of the two forces P and Q when the lever is in the state bordering on motion.

To determine the pressure Q: from any convenient scale, take Cp = the units of pressure in P=1 lb.; and construct the parallelogram of pressures $Cprq$; then the units in Cq will give the units of lbs. pressure in the force Q, that is in this case, Cq = ·77 lbs. nearly.

Art. 85.

Ex. 2. (See *fig.* p. 127. of the text.)

Here $OI = \sin 35° \times OT = \cdot5735 \times 1$
$= \cdot5735$;
$PI = PO - OI = 24 - \cdot5735 = 23\cdot4265$;
$QI = QO + OI = 3 + \cdot5735 = 3\cdot5735$;
P=60 lbs.; W=40 lbs.; and Q is required;

$$\therefore\ P \times PI = Q \times QI + W \times OI,$$
$$\therefore\ 60 \times 23{\cdot}4265 = Q \times 3{\cdot}5735 + 40 \times {\cdot}5735,$$
$$\therefore\ Q = 386 \text{ lbs.}$$

To determine generally the force of Traction.

ART. 86.

Ex. 1.

Here the general formula is

$$P = \frac{fW}{\cos b + f \sin b}.$$

In this example $W = 2$ cwts., $b = 35°$, and $f = {\cdot}7$; hence we have by substitution

$$P = \frac{{\cdot}7 \times 2}{\cos 35° + {\cdot}7 \times \sin 35°} = \frac{1{\cdot}4}{{\cdot}81915 + {\cdot}7 \times {\cdot}57357}$$
$$= 1{\cdot}14 \text{ cwts.}$$

Ex. 2.

In this case $b = 40°$, the other data remaining the same; hence we have

$$P = \frac{{\cdot}7 \times 2}{\cos 40° + {\cdot}7 \times \sin 40°} = 1{\cdot}19 \text{ cwts.}$$

To find the least Traction.

ART. 87.

Ex. 2.

Here we have the general formula, $P = W \sin a$. In this example $W = 9$ cwts., $a = 36° \ 30'$, and P is required;

$$\therefore\ P = 9 \times \sin 36° \ 30'$$
$$= 9 \times {\cdot}59482 = 5{\cdot}35428 \text{ cwts.}$$

General Proposition.

ART. 88.

Ex. 2.

Wt. in lbs. $= 1 \times 2240 = 2240$ lbs.
Resistance of friction $= \frac{1}{12}$ of 2240 lbs.
Space moved over in ft. $= \frac{1}{4} \times 5280 = 1320$ ft.
$\therefore$ Work of friction $= \frac{1}{12} \times 2240 \times 1320 = 246400$.
Work due to gravity $= 2240 \times 80 = 179200$.
$\therefore$ Total work $= 246400 + 179200 = 425600$.

Ex. 3.

In this case the work due to gravity must be subtracted from the work due to friction.

$\therefore$ Total work $= 246400 - 179200 = 67200$.

Equilibrium of the Arch.

Art. 91.

Ex. 3.

Here we have, as in Ex. 2.,

Wt. of the whole semi-arch $= 5100$ lbs.;

Dist. of its centre of gravity from the outer edge of the pier

$= 7{\cdot}6 - 1 = 6{\cdot}6$ ft.;

for it will be observed that the thickness of the pier is 1 foot less than it is in the preceding case;

Wt. whole pier $= 3 \times 1 \times 28{\cdot}5 \times 120 = 10260$ lbs.;
Moment whole pier $= 10260 \times 1\frac{1}{2} = 15390$;
Moment whole semi-arch $= 5100 \times 6{\cdot}6 = 33660$;
Moment horizontal thrust $= 1900 \times 28{\cdot}5 = 54150$.

Now the sum of the two first moments is less than the moment of the thrust: hence it follows that the pier will fall.

Ex. 4.

Let $x =$ the thickness of the pier; then

Distance centre of gravity of the semi-arch from the inner edge of the pier $= 7{\cdot}6 - 4 = 3{\cdot}6$ ft.;

$\therefore$ Distance of the same from the outer edge $= 3{\cdot}6 + x$;

Wt. whole pier $= x \times 1 \times 28{\cdot}5 \times 120$;

$\therefore$ Moment whole pier $= x \times 1 \times 28{\cdot}5 \times 120 \times \frac{x}{2} = 1710x^2$.

Wt. whole semi-arch $= 5100$;

$\therefore$ Moment whole semi-arch $= 5100 \times (3{\cdot}6 + x)$.

And, as in Ex. 2.,

Moment horizontal thrust $= 1900 \times 28{\cdot}5$.

Now since the pier is upon the point of overturning, the sum of the moments of the pier and the semi-arch will be equal to the moment of horizontal thrust,

$$\therefore\ 1710x^2 + 5100 \times (3{\cdot}6 + x) = 1900 \times 28{\cdot}5.$$

Solving this quadratic equation, we find

$$x = 3{\cdot}3 \text{ ft.},$$

which is the thickness of the pier required.

Ex. 5.

In order to determine the point of rupture with a sufficient degree of accuracy, the figure must be constructed on a scale of about four times the size of that of the accompanying drawing.

On K as a centre, with the radius KH $= 8$, sweep the semi-arch HDN; mark off NP $= {\cdot}75$, the height of the crown; take PE $= 9{\cdot}5$, and on E as a centre, with the radius EP, sweep the line of the extrados PVZ.

STRENGTH OF MATERIALS.

ART. 93.

Ex. 2.

$$\text{Area section} = 3 \times 3 = 9 \text{ sq. in.}$$

By the table we find,

The wt. supported by 1 sq. in. = 17207 lbs.;

$\therefore$ „ „ 9 sq. in. = 9 times 17207 lbs.

= 154863 lbs.

Ex. 4.

$$\text{Area section} = \tfrac{1}{2} \times \tfrac{1}{2} = \tfrac{1}{4} \text{ sq. in.}$$

By the table,

Wt. supported by 1 sq. in. = $25\frac{1}{2}$ tons;

$\therefore$ „ „ $\frac{1}{4}$ sq. in. = $\frac{1}{4}$ of $25\frac{1}{2}$ tons

= $6\frac{3}{8}$ tons.

Ex. 5.

$$\text{Area section rope} = (1\tfrac{1}{4})^2 \times \cdot 7854 = \frac{25}{16} \times \cdot 7854.$$

By the table,

Wt. supported by 1 sq. in. = 6400 lbs.

„ „ by $\frac{25}{16} \times \cdot 7854$ sq. in. $= 6400 \times \frac{25}{16} \times \cdot 7854$

$= 400 \times 25 \times \cdot 7854$

= 7854 lbs.

Ex. 7.

Let x = the breadth in inches; then

$$\text{Area section} = 2x \text{ sq. inches.}$$

By the table,

Wt. supported by 1 sq. in. of teak = 15000 lbs.

$\therefore$ „ „ by $2x$ „ „ $= 2x \times 15000$ lbs.;

But the weight which is to be supported is 30 tons, or 30×2240 lbs.;

$$\therefore\ 2x \times 15000 = 30 \times 2240,$$

$$x = \frac{30 \times 2240}{2 \times 15000} = 2{\cdot}24 \text{ in.}$$

Ex. 8.

Let $x =$ the diameter in inches; then

$$\text{Area section} = x^2 \times {\cdot}7854 \text{ sq. in.};$$

but from the table, 1 sq. in. of section will sustain $25\frac{1}{2}$ tons;

$$\therefore \text{ Wt. supported in tons} = x^2 \times {\cdot}7854 \times 25\tfrac{1}{2};$$

but, by the question, the weight supported is 50 tons;

$$\therefore\ x^2 \times {\cdot}7854 \times 25\tfrac{1}{2} = 50,$$

$$\therefore\ x = \sqrt{\frac{50}{{\cdot}7854 \times 25\frac{1}{2}}}$$

$$= \sqrt{2{\cdot}49} = 1{\cdot}58 \text{ inches nearly.}$$

Ex. 9.

No. lbs. supported by the rope $=$ diam.$^2 \times {\cdot}7854 \times 6400$.

But, by the question, the rope has to support 8 tons, or 8×2240 lbs.;

$$\therefore \text{ diam.}^2 \times {\cdot}7854 \times 6400 = 8 \times 2240,$$

$$\therefore \text{ diam.}^2 = \frac{8 \times 2240}{{\cdot}7854 \times 6400} = \frac{2{\cdot}8}{{\cdot}7854} = 3{\cdot}565,$$

$$\therefore \text{ diam.} = \sqrt{3{\cdot}565} = 1{\cdot}88 \text{ inches.}$$

Ex. 11.

No. sq. in. of section $= \left(\frac{3}{4}\right)^2 \times {\cdot}7854 = \frac{9}{16} \times {\cdot}7854$;

Wt. supported by the rope $=$ no. sq. in. of section $\times$ 6400 lbs.;

$\therefore$ Wt. supported by the rope $= \frac{9}{16} \times \cdot 7854 \times 6400$ lbs.

$$= 9 \times \cdot 7854 \times 400 = 2827 \cdot 44 \text{ lbs.};$$

Wt. 1 foot of rope 1 sq. in. of section $= \cdot 578$ lbs.;

Wt. of the whole rope $=$ no. inches of section $\times \cdot 578 \times$ no. feet length $= \frac{9}{16} \times \cdot 7854 \times \cdot 578 \times 200 \times 6$ lbs.

$$= 306 \cdot 4 + \text{ lbs.}$$

$\therefore$ Wt. coals $=$ wt. supported by the rope $-$ wt. of the rope itself $= 2827 \cdot 4 - 306 \cdot 4 = 2521$ lbs.

Ex. 12.

Let $x =$ the length of the rope in feet; then

Wt. 1 foot length of rope $=$ no. inches of section $\times \cdot 578$

$$= \frac{9}{16} \times \cdot 7854 \times \cdot 578;$$

$$\therefore \text{ Wt. of the whole rope} = \frac{9}{16} \times \cdot 7854 \times \cdot 578 \times x.$$

Wt. suspended by the rope in lbs. $= 1\frac{1}{4} \times 2240$

$$= 2800.$$

But by the last example the rope is capable of supporting 2827·4 lbs.,

$$\therefore \frac{9}{16} \times \cdot 7854 \times \cdot 578 \times x + 2800 = 2827 \cdot 4;$$

$$\therefore x = \frac{(2827 \cdot 4 - 2800) \times 16}{9 \times \cdot 7854 \times \cdot 578}$$

$$= \frac{438 \cdot 4}{4 \cdot 085} = 107 \cdot 4 \text{ ft.}$$

Ex. 13.

For the sake of simplicity, let us suppose that the rod is 1 inch in the section; then we have

Wt. 1 foot length of the rod $= \frac{1}{144}$ of 7700 oz. $= \frac{7700}{144 \times 16}$ lbs.

Now the rod is capable of supporting $25\frac{1}{2}$ tons, or $25\frac{1}{2} \times 2240$ lbs.;

$$\therefore \text{ Length rod in feet} = \frac{\text{wt. to break it}}{\text{wt. 1 ft. of length}}$$

$$= 25\tfrac{1}{2} \times 2240 \div \frac{7700}{144 \times 16}$$

$$= \frac{25\frac{1}{2} \times 2240 \times 144 \times 16}{7700} = 17091.$$

Ultimate Transverse Strength of Beams.

ART. 100.

Ex. 2.

Here the tabular number S is 12156 ; then by Rule 1., we have

$$W = \frac{12156 \times 3 \times 9^2}{6 \times 144} = 3418{\cdot}8 \text{ lbs.}$$

Ex. 3.

Here the tabular number S is 9336; then by Rule 1., we have

$$W = \frac{9336 \times 2 \times 6^2}{6 \times 8 \times 12} = 1167 \text{ lbs.}$$

Ex. 6.

Here the tabular number S is 10386; therefore by Rule 3., we have

$$W = \frac{4 \times 10386 \times 3 \times 6^2}{6 \times 9 \times 12} = 6924 \text{ lbs.}$$

Ex. 7.

Here the tabular number S is 10032, therefore by Rule 3., we have

$$W = \frac{4 \times 10032 \times 6 \times 12^2}{6 \times 18 \times 12} = 26752 \text{ lbs.}$$

Ex. 8.

Let x = the depth of the beam in inches; then, as in example 5., we have

$$W = \frac{4 \times 10386 \times 6 \times x^2}{6 \times 18 \times 12} \text{ (by cancelling)}$$

$$= \frac{10386 x^2}{18 \times 3}.$$

But, by the question, the breaking weight of the beam is to be 30000 lbs.;

$$\therefore \frac{10386 x^2}{18 \times 3} = 30000;$$

$$\therefore x^2 = \frac{30000 \times 18 \times 3}{10386} = 155{\cdot}97;$$

$$\therefore x = \sqrt{155{\cdot}97} = 12{\cdot}49 \text{ inches.}$$

Ex. 10.

Here $S = 10386$; therefore, by Rule 2., we have

$$W = \frac{2 \times 10386 \times 3 \times 6^2}{6 \times 10 \times 12} \text{ (by cancelling)}$$

$$= \frac{10386 \times 3}{10} = 3115{\cdot}8 \text{ lbs.}$$

Ex. 11.

Here $S = 10032$; therefore, by Rule 2., we have

$$W = \frac{2 \times 10032 \times 4 \times 9^2}{6 \times 12 \times 12} \text{ (by cancelling)}$$

$$= \frac{10032 \times 3}{4} = 7524 \text{ lbs.}$$

Ex. 13.

Here $S=9336$, the breadth $=6$ in., the depth $=12$ in., d the length $=18\times 12$ in.; therefore, by Rule 4., we have

$$W=\frac{8\times 9336\times 6\times 12^2}{6\times 18\times 12}\text{ (by cancelling)}$$

$$=\frac{8\times 9336\times 2}{3}=49792\text{ lbs.}$$

Ex. 14.

Let $x=$ the depth in inches; then, by Rule 4., we have

$$W=\frac{8\times 12156\times 4\times x^2}{6\times 20\times 12};$$

Wt. on each foot of the beam $=1000$ lbs.,

$\therefore$ Wt. on the whole beam $=1000\times 20=20000$ lbs.

But as the beam has just to sustain this weight, we have, equality,

$$\frac{8\times 12156\times 4\times x^2}{6\times 20\times 12}=20000;$$

$$\therefore x^2=\frac{20000\times 6\times 20\times 12}{8\times 12156\times 4}=74;$$

$$\therefore x=\sqrt{74}=8\cdot 6\text{ inches.}$$

Ex. 15.

Let $x=$ the distance in feet; now, as in Ex. 9., we have

$$W=\frac{2\times 6612\times 3\times 6^2}{6\times x\times 12}\text{ (by cancelling)}$$

$$=\frac{6612\times 3}{x}.$$

Wt. on each foot of the beam $=400$ lbs.,

Wt. on the whole beam $=400\times x$ lbs.

But when the beam is upon the point of breal this load, we have

$$\frac{6612\times 3}{x}=400\times x;$$

$$\therefore\ 400\,x^2=6612\times 3;$$

$$\therefore\ x^2=\frac{19836}{400}=49\cdot 59;$$

$$\therefore\ x=\sqrt{49\cdot 59}=7\cdot 04 \text{ ft.}$$

Ex. 16.

Let $x=$ the breadth of the beam in inches; tl Ex. 12., we have

$$W=\frac{8\times 12156\times x\times 9^2}{6\times 20\times 12}=5470\cdot 2\times x.$$

Wt. on each foot of the beam $=500$ lbs.,

$\therefore$ Wt. on the whole beam $=500\times 20=1000$

But when the beam is upon the point of breaking load, we have

$$5470\cdot 2\times x=10000;$$

$$\therefore\ x=\frac{10000}{5470\cdot 2}=1\cdot 828 \text{ inches.}$$

AILWAY CUTTINGS AND EMBANKMENTS.

ART. 102.

Ex. 2.

Iere, by the general formula, $g=32$, $q=66\times 2$, $p=3$, $=4$, $b=5$, $c=7$, $d=3$, $e=6$, and v, or the last perpenilar height, $=0$; hence we have

The content of the central part

$$=\frac{32\times 66\times 2}{2}\{4+0+2(5+7+3+6)\}=32\times 66\times 46$$
$$=97152.$$

Content of the two slopes

$$\frac{\times 66\times 2}{3}\{(4+5)^2+(5+7)^2+(7+3)^2+(3+6)^2+(6+0)^2$$
$$-(4\times 5+5\times 7+7\times 3+3\times 6+6\times 0)\}$$
$$6\times 2\{81+144+100+81+36-(20+35+21+18)\}$$
$$6\times 2\times 348=45936$$

$\therefore$ Content of whole cutting $=97152+45936$
$=143088$ cubic feet.

Ex. 3.

Iere $g=30$, $q=66$, $p=2$, $a=0$, $b=3$, $c=9$, $d=2$, 1, and $v=0$; hence we have

The content of the central part

$$\frac{0\times 66}{2}\{0+0+2(3+9+2+1)\}=30\times 33\times 30$$

9700 cubic feet.

Content of the two slopes

$$\frac{\times 66}{3}\{(0+3)^2+(3+9)^2+(9+2)^2+(2+1)^2+(1+0)^2$$
$$-(0\times 3+3\times 9+9\times 2+2\times 1+1\times 0)\}$$
$$\times 22\{9+144+121+9+1-(27+18+2)\}$$

0428

$\therefore$ Content of the whole cutting $= 29700 + 10428$
$= 40128$ c. ft.

Ex. 4.

Here the two extreme ordinates are 9 and 6, and the intermediate ones are 11, 12, 8, and 22; the width of the rail or $g = 30$ ft., $q = 66 \times 2$, and $p = 1\frac{1}{4}$; hence we have

The content of the central part

$$= \frac{30 \times 66 \times 2}{2}\{9 + 6 + 2(11 + 12 + 8 + 22)\}$$

$$= 30 \times 66 \times 121 = 239580 \text{ cubic feet.}$$

Content of the two slopes

$$= \frac{1\frac{1}{4} \times 66 \times 2}{3}\{(9 + 11)^2 + (11 + 12)^2 + (12 + 8)^2 + (8 + 22)^2$$
$$+ (22 + 6)^2 - (9 \times 11 + 11 \times 12 + 12 \times 8 + 8 \times 22 + 22 \times 6)\}$$
$$= 55\{400 + 529 + 400 + 900 + 784 - 635\}$$
$$= 130790$$

$\therefore$ Content of the whole cutting $= 130790 + 239580$
$= 370370$ c. ft.

Ex. 5.

Here the extreme ordinates are 5 and 4, and the intermediate ordinate is 6; the width of rail or $g = 30$, $q = 66 \times 2$, and $p = 2$; hence we have

The content of the central part

$$= \frac{30 \times 66 \times 2}{2}\{5 + 4 + 2 \times 6\} = 30 \times 66 \times 21$$

$$= 41580 \text{ cubic feet.}$$

Content of the two slopes

$$= \frac{2 \times 66 \times 2}{3}\{(5 + 6)^2 + (6 + 4)^2 - (5 \times 6 + 6 \times 4)\}$$

$$= 88\{121 + 100 - (30 + 24)\} = 14696 \text{ cubic feet.}$$

$\therefore$ Content of the whole cutting $= 14696 + 41580$
$= 56276$ cubic feet.

Ex. 6.

In this case, the extreme ordinates are 0 and 0, and the intermediate ordinates are 10, 30, and 40; the width of the rail or $g = 30$, $q = 66 \times 2$, and $p = 4$; hence we have

The content of the central part

$$= \frac{30 \times 66 \times 2}{2}\{0 + 0 + 2(10 + 30 + 40)\}$$

$$= 30 \times 66 \times 160 = 316800 \text{ cubic feet.}$$

Content of the two slopes

$$= \frac{4 \times 66 \times 2}{3}\{(0 + 10)^2 + (10 + 30)^2 + (30 + 40)^2 + (40 + 0)^2$$

$$-(0 \times 10 + 10 \times 30 + 30 \times 40 + 40 \times 0)\}$$

$$= 176\{100 + 1600 + 4900 + 1600 - (0 + 300 + 1200 + 0)\}$$

$$= 6700 \times 176 = 1179200 \text{ c. ft.}$$

$\therefore$ Content of the whole cutting $= 1179200 + 316800$

$= 1496000$ cubic feet.

Ex. 7.

Time with each full load $= \frac{5}{10} = \frac{1}{2}$ hours.

Time in returning with empty waggons $= \frac{5}{20} = \frac{1}{4}$ hours.

$\therefore$ Time each journey $= \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$ hours.

No. journeys per day $= 10 \div \frac{3}{4} = \frac{40}{3}$.

No. c. ft. conveyed each journey $= 400$,

$\therefore$ „ „ „ per day $= 400 \times \frac{40}{3} = \frac{16000}{3}$.

Now a shoveller lifts 500 c. ft. per day;

$$\therefore \text{ No. shovellers} = \frac{16000}{3} \div 500 = \frac{16000}{3 \times 500}$$

$$= \frac{32}{3} = 10\tfrac{2}{3}.$$

Now as the material requires 2 pickmen to 3 shovellers, we have

$$\text{No. pickmen} = \tfrac{2}{3} \text{ of } 10\tfrac{2}{3} = \tfrac{2}{3} \text{ of } \frac{32}{3}$$

$$= \frac{64}{9} = 7\tfrac{1}{9}.$$

$$\text{Total no. days' work of shovellers} = \frac{1496000}{500}$$

$$= 2992.$$

$$\therefore \text{ Total no. days' work of pickmen} = \tfrac{2}{3} \text{ of } 2992$$

$$= 1994\tfrac{2}{3}.$$

$$\therefore \text{ Total no. days' work} = 2992 + 1994\tfrac{2}{3}$$

$$= 4986\tfrac{2}{3}.$$

$$\therefore \text{ Cost} = 4986\tfrac{2}{3} \times 3s. = 14960s.$$

$$= £748.$$

Ex. 8.

Here the extreme ordinates are 0 and 0, and the intermediate ones are 20, 30, 60, and 40; the width of the rail or $g = 30$, the distance between each perpendicular, or $q = 66$, and the ratio of the slope, or $p = 1$; hence we have, by the formulæ,

The content of the central part

$$= \frac{30 \times 66}{2}\{0 + 0 + 2(20 + 30 + 60 + 40)\}$$

$$= 30 \times 66 \times 150 = 297000 \text{ c. ft.}$$

Content of the two slopes

$$= \frac{1 \times 66}{3}\{(0 + 20)^2 + (20 + 30)^2 + (30 + 60)^2 + (60 + 40)^2$$
$$+ (40 + 0)^2 - (0 \times 20 + 20 \times 30 + 30 \times 60 + 60 \times 40 + 40 \times 0)\}$$

$$= 22\{400 + 2500 + 8100 + 10000 + 1600 - (600 + 1800$$
$$+ 2400)\}$$

$$= 391600.$$

$$\therefore \text{ Content of the whole cutting} = 391600 + 297000$$
$$= 688600 \text{ cubic feet.}$$

$$\text{Total no. days' work of shovellers} = \frac{688600}{500}$$
$$= 1377\tfrac{1}{5}.$$

$$\text{Total no. days' work of pickmen} = \tfrac{2}{3} \text{ of } 1377\tfrac{1}{5}$$
$$= 918\tfrac{2}{15}.$$

$$\therefore \text{ Total no. days' work} = 1377\tfrac{1}{5} + 918\tfrac{2}{15}$$
$$= 2295\tfrac{1}{3}$$

$$\therefore \text{ Cost} = 2295\tfrac{1}{3} \times 3s. = 6886s.$$
$$= £344 \ 6s. \ 0d.$$

EXAMPLES FOR PUPILS.

The number prefixed to each set of examples refers to the article in the text where the principle involved in the questions is explained.

Art. 2.

Ex. 1.

Work in raising 1 lb. to the height of 15 ft. = 15;
∴ „ „ 7 lb. „ „ of 15 ft. = 15 × 7 = 105.

Ex. 2.

Wt. in lbs. = 3 × 112 = 336 lbs.
Work in raising 1 lb. to the height of 5 ft. = 5;
∴ „ „ 336 lb. „ „ of 5 ft. = 336 × 5
= 1680.

Ex. 3.

Weight of a c. ft. of water = 62·5 lbs.;
Work in raising 1 c. ft. of water to the height of 36 feet
= 36 × 62·5 = 2250.

Ex. 4.

Weight of a c. ft. of water = 62·5 lbs.;
∴ Work in raising this water to the height of 24 feet
= 62·5 × 24 = 1500.

Ex. 5.

Here the units of work to be performed are 35;

∴ Height to which 1 lb. is raised = 35 ft.

„ „ 7 lbs. „ $= \frac{35}{7} = 5$ ft.

Or thus.

Let $x =$ the height in feet; then

the work $= 7 \times x$.

But this work is equal to 35 units;

$$\therefore\ 7 \times x = 35,$$

$$\therefore\ x = \frac{35}{7} = 5 \text{ feet.}$$

Ex. 6.

Weight of a c. ft. of water $= 62{\cdot}5$ lbs.

∴ „ 6 c. ft. „ $= 62{\cdot}5 \times 6 = 375$ lbs.

∴ Work in raising 375 lbs. to the height of 8 feet

$$= 375 \times 8 = 3000.$$

Art. 3.

Ex. 1.

Weight of water in lbs. $= 87 \times 62{\cdot}5 = 5437{\cdot}5$ lbs.

Depth of the pit in ft. $= 60 \times 6 = 360$ ft.

∴ Work done per min. $= 360 \times 5437{\cdot}5 = 1957500$

$$\therefore\ \text{H. P.} = \frac{1957500}{33000} = 59{\cdot}3.$$

Ex. 2.

Weight of water in lbs. $= 87 \times 62{\cdot}5 = 5437{\cdot}5$.

Depth of pit in ft. $= 25 \times 6 = 150$;

$\therefore$ Work done per min. $= 5437{\cdot}5 \times 150 = 815625$

$$\therefore \text{ H. P.} = \frac{815625}{33000} = 24{\cdot}7.$$

Ex. 3.

Work engine per min. $= 50 \times 33000 = 1650000$,

Depth in ft. $= 90 \times 6$;

$\therefore$ Work in raising 1 c. ft. water $= 62{\cdot}5 \times 90 \times 6 = 33750$;

$$\therefore \text{ No. ft. raised per min.} = \frac{1650000}{33750} = 48{\cdot}8.$$

Or thus.

Let $x =$ no. c. ft. water raised per min.,

Wt. water $= 62{\cdot}5 \times x$;

$\therefore$ Work in raising water per min. $= 62{\cdot}5 \times x \times 90 \times 6$

$= x \times 33750$;

but the work of the engine per min. is 1650000;

$$\therefore \; x \times 33750 = 1650000;$$

$$\therefore \; x = \frac{1650000}{33750} = 48{\cdot}8 \text{ c. ft.}$$

Ex. 4.

Work engine per min. $= 10 \times 33000 = 330000$;

Work in raising 1 c. ft. water $= 62{\cdot}5 \times 90 \times 6 = 33750$;

$$\therefore \text{ No. ft. raised per min.} = \frac{330000}{33750} = 9{\cdot}7.$$

Ex. 5.

Work engine per hour $= 3 \times 33000 \times 60$.

Depth of the pit in feet $= 120 \times 6$.

Work in raising 1 ton of coals $= 2240 \times 120 \times 6$;

$$\therefore \text{ No. of tons} = \frac{3 \times 33000 \times 60}{2240 \times 120 \times 6}$$

$$= \frac{33000 \times 10}{2240 \times 40} = \frac{330000}{89600} = 3{\cdot}6.$$

Or thus.

Let x = no. tons of coals raised per hour;

Wt. coals in lbs. $= x \times 2240$;

$\therefore$ Work in raising coals $= x \times 2240 \times 120 \times 6$;

but the work of the engine per hour $= 3 \times 33000 \times 60$;

$$\therefore x \times 2240 \times 120 \times 6 = 3 \times 33000 \times 60,$$

$$\therefore x = \frac{3 \times 33000 \times 60}{2240 \times 120 \times 6} = 3{\cdot}6 \text{ tons.}$$

Ex. 6.

Work engine per hour $= 3 \times 33000 \times 60$.

Depth of the pit in feet $= 180 \times 6$;

$\therefore$ Work in raising 1 ton of coals $= 2240 \times 180 \times 6$;

$$\therefore \text{ No. of tons} = \frac{3 \times 33000 \times 60}{2240 \times 180 \times 6} \text{ (by cancelling)}$$

$$= \frac{33000}{2240 \times 6} = \frac{33000}{13440} = 2{\cdot}4.$$

Ex. 7.

Weight of coals in lbs. $= 14 \times 2240$.

Depth of mine in feet $= 80 \times 6$;

$$\therefore \text{ Work per min.} = \frac{14 \times 2240 \times 80 \times 6}{60};$$

$$\text{And H. P.} = \frac{14 \times 2240 \times 80 \times 6}{60 \times 33000} = \frac{14 \times 2240 \times 8}{33000} = 7{\cdot}6.$$

Ex. 8.

Weight of coals in lbs. $= 14 \times 2240$.

Depth of mine in ft. $= 60 \times 6$;

$$\therefore \text{ Work per min.} = \frac{14 \times 2240 \times 60 \times 6}{60} = 14 \times 2240 \times 6;$$

$$\therefore \text{ H. P.} = \frac{14 \times 2240 \times 6}{33000} = 5{\cdot}7.$$

Ex. 9.

Work engine per min. $= 20 \times 33000 = 660000$;
Wt. water in lbs. $= 40 \times 62{\cdot}5 = 2500$ lbs.;
Work in raising 40 c. ft. of water 1 ft. $= 2500 \times 1 = 2500$;

$$\therefore \text{ No. ft. in depth} = \frac{660000}{2500} = 264 \text{ feet, or } 44 \text{ fathoms.}$$

Ex. 10.

Work engine per min. $= 15 \times 33000 = 495000$;
Wt. water in lbs. $= 40 \times 62{\cdot}5 = 2500$ lbs.;
Work in raising 40 c. ft. of water 1 ft. $= 2500 \times 1$;

$$\therefore \text{ No. ft. in depth} = \frac{495000}{2500 \times 1} = 198 \text{ ft. or } 33 \text{ fathoms.}$$

Ex. 11.

Work of engine per min. $= 40 \times 33000$;
Wt. water in lbs. $= 4000 \times 62{\cdot}5$;
Mean height in ft. $= 60 \times 6$;
$\therefore$ Work in raising water $= 4000 \times 62{\cdot}5 \times 60 \times 6$;

$$\therefore \text{ No. of min. to raise the water} = \frac{4000 \times 62{\cdot}5 \times 60 \times 6}{40 \times 33000}$$

$$= \frac{62{\cdot}5 \times 6 \times 2}{11} = 68 \text{ min., or } 1 \text{ h. } 8 \text{ min.}$$

Or thus.

Let x = no. minutes;
Work engine in x min. $= 40 \times 33000 \times x$;
Work in raising water $= 4000 \times 62{\cdot}5 \times 60 \times 6$;

$$\therefore\ 40\times 33000\times x=4000\times 62{\cdot}5\times 60\times 6;$$

$$\therefore\ x=\frac{4000\times 62{\cdot}5\times 60\times 6}{40\times 33000}=68\text{ min.}+.$$

Ex. 12.

Work of engine per min. $=40\times 33000$;

Work in raising water $=4000\times 62{\cdot}5\times 15\times 6$;

$$\therefore\ \text{No. of men to raise the water}=\frac{4000\times 62{\cdot}5\times 15\times 6}{40\times 33000}$$

$$=\frac{62{\cdot}5\times 15\times 2}{110}=17\text{ min.}+.$$

Ex. 13.

Wt. coals in lbs. $=4\times 112=448$ lbs.;

Depth pit in ft. $=80\times 6=480$ ft.;

$$\therefore\ \text{Work in raising coals}=448\times 480;$$

but this work is done in 4 minutes,

$$\therefore\ \text{Work per min.}=\frac{448\times 480}{4}$$

$$=448\times 120=53760.$$

$$\therefore\ \text{H. P.}=\frac{53760}{33000}=1{\cdot}6.+.$$

Ex. 14.

$$\text{Work done per min.}=\frac{4\times 112\times 100\times 6}{4};$$

$$\therefore\ \text{H. P.}=\frac{4\times 112\times 100\times 6}{4\times 33000}$$

$$=\frac{112\times 2}{110}=\frac{224}{110}=2.$$

ART. 5.

Ex. 1.

By the table, p. 9. of the text, the work of a man per minute is 1126;

$$\therefore \text{ Work per day} = 1126 \times 60 \times 6.$$

Work in raising 1 sack $= 2\frac{1}{2} \times 112 \times 30$;

$$\therefore \text{ No. of sacks} = \frac{1126 \times 60 \times 6}{2\frac{1}{2} \times 112 \times 30}$$

$$= \frac{1126 \times 2 \times 3}{2\frac{1}{2} \times 56} = \frac{6756}{140} = 48 \cdot 2 +.$$

Or thus.

Let $x =$ the no. sacks per day.

Work in raising 1 sack $= 2\frac{1}{2} \times 112 \times 30$;

$$\therefore \text{ Work in raising } x \text{ sacks} = 2\frac{1}{2} \times 112 \times 30 \times x.$$

But this must be equal to the work of the man per day;

$$\therefore \ 2\frac{1}{2} \times 112 \times 30 \times x = 1126 \times 60 \times 6;$$

$$\therefore \ x = \frac{1126 \times 60 \times 6}{2\frac{1}{2} \times 112 \times 30} = 48 +.$$

Ex. 2.

Work per day $= 1126 \times 60 \times 6$.

Work in raising 1 sack $= 2\frac{1}{2} \times 112 \times 25$;

$$\therefore \text{ No. of sacks} = \frac{1126 \times 60 \times 6}{2\frac{1}{2} \times 112 \times 25} = 58 \text{ nearly.}$$

Ex. 3.

By the table, p. 10. of the text, the work of the man per minute is 470;

$$\therefore \text{ Work per day} = 470 \times 60 \times 10.$$

Wt. earth $= 1692 \times 100$;

$$\therefore \text{ Work in raising earth} = 1692 \times 100 \times 5;$$

$$\therefore \text{ No. days} = \frac{1692 \times 100 \times 5}{470 \times 60 \times 10}$$

$$= \frac{846000}{282000} = 3 \text{ days.}$$

Ex. 4.

Work per day $= 470 \times 60 \times 10$;

Work in raising earth $= 5076 \times 100 \times 5$;

$$\therefore \text{ No. of days} = \frac{5076 \times 100 \times 5}{470 \times 60 \times 10} = 9.$$

Or thus.

Let $x =$ the no. days.

Work per day $= 470 \times 60 \times 10$;

$\therefore$ Work in x days $= 470 \times 60 \times 10 \times x$.

But this is equal to the work requisite to raise the earth;

$$\therefore 470 \times 60 \times 10 \times x = 5076 \times 100 \times 5;$$

$$\therefore x = \frac{5076 \times 100 \times 5}{470 \times 60 \times 10} = 9.$$

Ex. 5.

Here, by the table, p. 9. of the text, the work of a man per min., working for 8 hours per day, is 2560;

$\therefore$ Work per day $= 2560 \times 60 \times 8$.

Depth pit in ft. $= 20 \times 6$;

$\therefore$ Work in raising 1 ton $= 2240 \times 20 \times 6$;

$$\therefore \text{ No. of tons} = \frac{2560 \times 60 \times 8}{2240 \times 20 \times 6}$$

$$= \frac{2560 \times 10}{280 \times 20} = 4{\cdot}6 \text{ nearly.}$$

Ex. 6.

Work per day $= 2560 \times 60 \times 8$;

Depth pit in ft. $= 28 \times 6$;

$$\therefore \text{ Work in raising 1 ton} = 2240 \times 28 \times 6;$$

$$\therefore \text{ No. of tons} = \frac{2560 \times 60 \times 8}{2240 \times 28 \times 6}$$

$$\text{(by cancelling)} = \frac{640 \times 10}{280 \times 7} = 3{\cdot}3 \text{ nearly.}$$

Or thus.

Let $x =$ the no. of tons raised.

Work in raising 1 ton $= 2240 \times 28 \times 6$;

$\therefore$ Work in raising x tons $= 2240 \times 28 \times 6 \times x$.

But this is equal to the work which the man performs per day;

$$\therefore 2240 \times 28 \times 6 \times x = 2560 \times 60 \times 8;$$

$$\therefore x = \frac{2560 \times 60 \times 8}{2240 \times 28 \times 6} = 3{\cdot}3 \text{ nearly.}$$

Ex. 7.

The work done by a man per min. in this case is 1560;

$\therefore$ Work per day $= 1560 \times 60 \times 6$.

Work in raising 1 ton $= 2240 \times 20 \times 6$;

$$\therefore \text{ No. of tons} = \frac{1560 \times 60 \times 6}{2240 \times 20 \times 6}$$

$$= \frac{1560 \times 3 \times 1}{2240 \times 1 \times 1} = \frac{4680}{2240} = 2{\cdot}09.$$

Ex. 8.

Wt. water in lbs. $= 62{\cdot}5 \times 400 = 25000$ lbs.;

Mean height to which the water is raised $= 30$ ft.;

$\therefore$ Work in pumping water $= 25000 \times 30 = 750000$.

But in this case a man performs 2600 units of work in a minute;

$$\therefore \text{ No. min. in raising the water} = \frac{\text{work in raising water}}{\text{work of man per min.}}$$

$$= \frac{750000}{2600} = 288 \text{ min., or 4 h. 48 min.}$$

Ex. 9.

Here work in pumping water $= 25000 \times 10 = 250000$;

$$\therefore \text{ No. min. in raising the water} = \frac{250000}{2600} = 96 + \text{ min.}$$
$$= 1 \text{ h. } 36 \text{ min.}$$

Ex. 10.

Work man per day $= 2600 \times 60 \times 8$;
Depth in ft. $= 20 \times 6$;

$\therefore$ Work in raising 1 c. ft. of water $= 62{\cdot}5 \times 20 \times 6$;

$$\therefore \text{ No. c. ft. water} = \frac{2600 \times 60 \times 8}{62{\cdot}5 \times 20 \times 6}$$
$$= \frac{2600 \times 10 \times 8}{62{\cdot}5 \times 20} = \frac{208000}{1250} = 166{\cdot}4.$$

Or thus.

Let $x =$ no. c. ft. of water pumped.
Wt. water $= 62{\cdot}5 \times x$;
Work in pumping water $= 62{\cdot}5 \times x \times 20 \times 6$.

But this is equal to the work which the man performs in a day;

$$\therefore 62{\cdot}5 \times x \times 20 \times 6 = 2600 \times 60 \times 8;$$
$$\therefore x = \frac{2600 \times 60 \times 8}{62{\cdot}5 \times 20 \times 6} = 166{\cdot}4.$$

Ex. 11.

Work of 2 men per day $= 2600 \times 2 \times 60 \times 8$;
Wt. ram in lbs. $= 5 \times 112 = 560$ lbs.;

$\therefore$ Work in each lift of the ram $= 560 \times 12$;

$$\therefore \text{ No. strokes} = \frac{2600 \times 2 \times 60 \times 8}{560 \times 12} = 371{\cdot}4.$$

Ex. 12.

Work of 2 men per day $= 2600 \times 2 \times 60 \times 8$;
Work in each lift of the ram $= 5 \times 112 \times 16$;

$$\therefore \text{ No. of strokes} = \frac{2600 \times 2 \times 60 \times 8}{5 \times 112 \times 16}$$

$$= \frac{2600 \times 60}{5 \times 112} = \frac{156000}{560} = 278{\cdot}5+.$$

Or thus.

Let $x =$ the no. of strokes.

Work in each lift of the ram $= 5 \times 112 \times 16$;

$\therefore$ Work in x lifts of the ram $= 5 \times 112 \times 16 \times x$.

But this is equal to the work performed by the two men per day;

$$\therefore 5 \times 112 \times 16 \times x = 2600 \times 2 \times 60 \times 8;$$

$$\therefore x = \frac{2600 \times 2 \times 60 \times 8}{5 \times 112 \times 16} = 278{\cdot}5.$$

ART. 7.

Ex. 1.

Pressure to move 1 ton $= 6$ lbs.;

$\therefore$ Pressure to move 45 tons $= 45$ times 6 lbs.

$= 270$ lbs.

Ex. 2.

Here the resistance is 270 lbs., and the space over which it is moved is 9 feet;

$\therefore$ Work due to friction $= 270 \times 9 = 2430$

Ex. 3.

Resistance of friction $= 80 \times 6 = 480$ lbs.;

$$\text{Distance moved in ft. per min.} = \frac{20 \times 5280}{60} = 1760 \text{ ft.};$$

$\therefore$ Work due to friction $= 480 \times 1760$;

$$\text{And H.P.}=\frac{480\times1760}{33000}$$
$$=\frac{160\times1760}{11000}=25{\cdot}6.$$

Ex. 4.

Resistance of friction $=60\times6=360$ lbs.;

Distance moved in ft. per min. $=\dfrac{20\times5280}{60}=1760$ ft.;

$\therefore$ Work due to friction $=360\times1760$;

$$\text{And H.P.}=\frac{60\times1760}{33000}$$
$$=\frac{120\times1760}{11000}=19{\cdot}2.$$

Ex. 5.

Resistance of friction $=7\times100=700$ lbs.;

Speed per min. $=\dfrac{15\times5280}{60}=15\times88=1320$ ft.;

$\therefore$ Work due to friction $=700\times1320$;

$$\text{And H.P.}=\frac{700\times1320}{33000}$$
$$=\frac{700\times440}{11000}=\frac{308}{11}=28.$$

Ex. 6.

Resistance of friction $=100\times5\tfrac{1}{2}=550$ lbs.;

Speed per min. $=\dfrac{15\times5280}{60}=1320$ ft.;

$\therefore$ Work due to friction $=1320\times550$;

$$\text{And H.P.}=\frac{1320\times550}{33000}$$
$$=\frac{440\times550}{11000}=\frac{440\times50}{1000}=22.$$

Ex. 7.

Work of engine per hour $= 30 \times 33000 \times 60$;

Space moved over in feet per hour $= 40 \times 5280$;

$\therefore$ Work consumed per hour in moving 1 ton $= 7 \times 40 \times 5280$;

$$\therefore \text{ No. of tons} = \frac{30 \times 33000 \times 60}{7 \times 40 \times 5280}$$

$$= \frac{30 \times 33000}{7 \times 40 \times 88} = \frac{990000}{24640} = 40 \cdot 1 + .$$

Or thus.

Let $x =$ the no. tons.

Work due to friction per hour $= x \times 7 \times 40 \times 5280$.

But when the train moves uniformly, this is equal to the work of the engine per hour;

$$\therefore x \times 7 \times 40 \times 5280 = 30 \times 33000 \times 60;$$

$$\therefore x = \frac{30 \times 33000 \times 60}{7 \times 40 \times 5280} = 40 \cdot 1.$$

Ex. 8.

Work of engine per hour $= 25 \times 33000 \times 60$;

Work consumed per hour in moving 1 ton $= 7 \times 40 \times 5280$;

$$\therefore \text{ No. of tons} = \frac{25 \times 33000 \times 60}{7 \times 40 \times 5280} = 33 \cdot 4.$$

Ex. 9.

Work done by engine per hour $= 70 \times 33000 \times 60$;

Resistance friction $= 6 \times 80$ lbs.;

$\therefore$ Work in moving the train 1 mile $= 6 \times 80 \times 5280$;

$$\therefore \text{ No. miles per hour} = \frac{70 \times 33000 \times 60}{6 \times 80 \times 5280}$$

$$= \frac{70 \times 33000 \times 10}{80 \times 5280} = \frac{23100000}{422400} = 54 \cdot 6 + .$$

Or thus.

Let $x =$ the no. miles per hour; then
Speed per hour in feet $= x \times 5280$;
Resistance friction in lbs. $= 6 \times 80$ lbs.;
$\therefore$ Work due to friction per hour $= 6 \times 80 \times x \times 5280$.
But this is equal to the work of the engine per hour;

$$\therefore\ 6 \times 80 \times x \times 5280 = 70 \times 33000 \times 60;$$

$$\therefore\ x = \frac{70 \times 33000 \times 60}{6 \times 80 \times 5280} = 54{\cdot}6.$$

Ex. 10.

Work done by engine per hour $= 70 \times 33000 \times 60$;
Resistance friction $= 6 \times 100$ lbs.;
$\therefore$ Work in moving the train 1 mile $= 6 \times 100 \times 5280$

$$\therefore\ \text{No. miles per hour} = \frac{70 \times 33000 \times 60}{6 \times 100 \times 5280}$$

$$= \frac{70 \times 33000 \times 10}{100 \times 5280} = \frac{23100000}{528000} = 43{\cdot}75.$$

Ex. 11.

Work done by engine per hour $= 60 \times 33000 \times 60$;
Resistance friction $= 6 \times 120$ lbs.;
$\therefore$ Work in moving train 1 mile $= 6 \times 120 \times 5280$;

$$\therefore\ \text{No. of miles per hour} = \frac{60 \times 33000 \times 60}{6 \times 120 \times 5280}$$

$$= \frac{33000 \times 10}{120 \times 88} = \frac{330000}{10560} = 31{\cdot}25.$$

Ex. 12.

Work done by engine per hour $= 60 \times 33000 \times 60$;
Resistance friction $= 8 \times 120$ lbs.;
$\therefore$ Work in moving train 1 mile $= 8 \times 120 \times 5280$;

$$\therefore \text{ No. of miles per hour} = \frac{60 \times 33000 \times 60}{8 \times 120 \times 5280}$$

$$= \frac{33000}{8 \times 2 \times 88} = \frac{33000}{1408} = 23{\cdot}4.$$

Ex. 13.

Resistance friction $= 90 \times 7$ lbs.;

Space moved over in feet $= 80 \times 5280$;

$\therefore$ Work expended in moving the train 80 miles

$$= 80 \times 5280 \times 90 \times 7.$$

Work of engine per hour $= 100 \times 33000 \times 60$;

$$\therefore \text{ No. of hours} = \frac{80 \times 5280 \times 90 \times 7}{100 \times 33000 \times 60}$$

$$= \frac{80 \times 88 \times 3 \times 7}{100 \times 1100} = \frac{14{\cdot}7840}{11{\cdot}0000} = 1{\cdot}34+.$$

Ex. 14.

Here, resistance friction $= 45 \times 7$ lbs.;

$\therefore$ Work expended in mov. train 80 m. $= 80 \times 5280 \times 45 \times 7$.

Work of engine per hour $= 100 \times 33000 \times 60$;

$$\therefore \text{ No. of hours} = \frac{80 \times 5280 \times 45 \times 7}{100 \times 33000 \times 60} \text{ (by cancelling)}$$

$$= \frac{80 \times 8 \times 15 \times 7}{100 \times 1000} = \frac{67200}{100000} = {\cdot}672.$$

Ex. 15.

Resistance friction, or traction horse $= \frac{1}{20}$ of 2240 lbs.

$= 112$ lbs.;

$\therefore$ Work horse $= 112 \times 260 = 29120$.

Ex. 16.

Resistance friction $= \frac{1}{30}$ of 2240 lbs.

$$= \frac{2240}{30} \text{ lbs.};$$

$$\therefore \text{ Work horse} = \frac{2240}{30} \times 260 = 19413.$$

ART. 9.

Ex. 1.

Work in cutting 1 sq. ft. $= 29000$;

$\therefore$ Work in cutting 4000 sq. ft. $= 29000 \times 4000$;

But this work is done in 10 hours;

$$\therefore \text{ Work per min.} = \frac{29000 \times 4000}{60 \times 10};$$

$$\text{And H. P.} = \frac{29000 \times 4000}{60 \times 10 \times 33000}$$

$$= \frac{29 \times 20}{3 \times 33} = \frac{580}{99} = 5{\cdot}8 \text{ nearly.}$$

Ex. 2.

Work in cutting 1 sq. ft. $= 29000$;

$\therefore$ „ „ 5000 sq. ft. $= 29000 \times 5000$.

$$\text{Work per min.} = \frac{29000 \times 5000}{60 \times 10};$$

$$\therefore \text{ H. P.} = \frac{29000 \times 5000}{60 \times 10 \times 33000}$$

$$= \frac{1450}{198} = 7{\cdot}3.$$

Ex. 3

Work of engine per day $= 3 \times 33000 \times 60 \times 12$;

Work in cutting 1 sq. ft. $= 29000$;

$$\therefore \text{ No. of sq. ft.} = \frac{3 \times 33000 \times 60 \times 12}{29000} = 2457.$$

Ex. 4.

Work of engine per day $= 3 \times 33000 \times 60 \times 10$;

Work in cutting 1 sq. ft. $= 29000$;

$$\therefore \text{ No. of sq. ft.} = \frac{3 \times 33000 \times 10 \times 60}{29000} = 2048 \text{ ft.}$$

Or thus.

Let $x=$ the no. of sq. ft. cut by the engine; then
Work in cutting 1 sq. ft. $=29000$;

$$\therefore \text{ Work in cutting } x \text{ sq. ft.} = 29000\times x.$$

But this is equal to the work performed by the engine;

$$\therefore\ 29000\times x=3\times 33000\times 60\times 10;$$

$$\therefore\ x=\frac{3\times 33000\times 60\times 10}{29000}=2048 \text{ sq. ft.}$$

Ex. 5.

Work of engine per min. $=9\times 33000$;
Work in cutting timber $=6000\times 29000$;

$$\therefore \text{ No. of min.}=\frac{6000\times 29000}{9\times 33000}$$

$$=\frac{2\times 29000}{9\times 11}=\frac{58000}{99}=585\cdot 8+\text{ min., or } 9\cdot 76 \text{ hours.}$$

Or thus.

Let $x=$ no. hours; then
Work engine $=9\times 33000\times 60\times x$.
But this must be equal to the work in cutting the timber;

$$\therefore\ 9\times 33000\times 60\times x=6000\times 29000;$$

$$\therefore\ x=\frac{6000\times 29000}{9\times 33000\times 60}$$

$$=\frac{29000}{9\times 330}=9\cdot 76 \text{ hours.}$$

Ex. 6.

Work of engine per min. $=10\times 33000$;
Work in cutting timber $=6000\times 29000$;

$$\therefore \text{ No. of min.}=\frac{6000\times 29000}{10\times 33000}$$

$$=\frac{2\times 29000}{10\times 11}=\frac{58000}{110}=527\cdot 2+\text{min., or } 8\cdot 78 \text{ hours.}$$

ART. 11.

Ex. 1.

Speed of train per min. $= \frac{20 \times 5280}{60} = 1760$ ft.;

Weight of train in lbs. $= 60 \times 2240 = 134400$;

Rise of rail per ft. $= \frac{1}{100}$;

$\therefore$ Rise of rail per 1760 ft. $= 1760 \times \frac{1}{100} = 17{\cdot}6$;

$\therefore$ Work due to gravity per min. $= 134400 \times 17{\cdot}6$
$= 2365440$;

$$\therefore \text{ H. P.} = \frac{2365440}{33000} = 71{\cdot}6.$$

Ex. 2.

Speed of train per min. $= \frac{15 \times 5280}{60} = 1320$ ft.;

Weight of train in lbs. $= 60 \times 2240 = 134400$ lbs.;

Rise of rail per foot $= \frac{1}{100}$;

$\therefore$ Rise of rail per 1320 ft. $= 1320 \times \frac{1}{100} = 13{\cdot}2$ ft.;

$\therefore$ Work due to gravity per min. $= 134400 \times 13{\cdot}2$.;

$$\text{And H. P.} = \frac{134400 \times 13{\cdot}2}{33000} = 53{\cdot}7.$$

Ex. 3.

Speed of train per min. $= \frac{20 \times 5280}{60} = 1760$ ft.;

Weight of train in lbs. $= 60 \times 2240 = 134400$ lbs.;

Rise of rail per ft. $= \frac{1}{100}$;

$\therefore$ Rise of rail in 1760 ft. $= \frac{1}{100} \times 1760 = 17{\cdot}6$ ft.;

$\therefore$ Work due to gravity per min. $= 17{\cdot}6 \times 134400$
$= 2365440$;

„ „ friction per min. $= 6 \times 60 \times 1760$
$= 633600$;

$\therefore$ Total work per min. $= 2999040$;

And H. P. $= \frac{2999040}{33000} = 90{\cdot}8$.

Ex. 4.

Speed of train per min. $= \frac{40 \times 5280}{33000} = 3520$ ft.;

Weight of train in lbs. $= 75 \times 2240 = 168000$ lbs.;

Rise of rail per ft. $= \frac{1}{8} \div 100 = \frac{1}{800}$;

$\therefore$ „ „ 3520 ft. $= \frac{1}{800} \times 3520 = 4{\cdot}4$ ft.;

$\therefore$ Work due to gravity per min. $= 4{\cdot}4 \times 168000 = 739200$;

„ „ friction per min. $= 6 \times 75 \times 3520$
$= 1584000$;

$\therefore$ Total work per min. $= 739200 + 1584000 = 2323200$;

And H. P. $= \frac{2323200}{33000} = 70{\cdot}4$.

Ex. 5.

Speed of train per min. $= \frac{40 \times 5280}{60} = 3520$ ft.;

Weight of train in lbs. $= 75 \times 2240 = 168000$ lbs.;

Rise of rail per ft. $= \frac{1}{3} \div 100 = \frac{1}{300}$ ft.;

$\therefore$ „ „ 3520 ft. $= \frac{1}{300} \times 3520 = 11{\cdot}73$ ft.;

$\therefore$ Work due to gravity per min. $= 168000 \times 11{\cdot}73$
$= 1970640$;

" " friction per min. $= 6 \times 75 \times 3520$
$= 1584000$;

$\therefore$ Total work per min. $= 1584000 + 1970640 = 3554640$;

$$\text{And H. P.} = \frac{3554640}{33000} = 107{\cdot}7.$$

Ex. 6.

$$\text{Speed of train per min.} = \frac{40 \times 5280}{60} = 3520 \text{ ft.};$$

Weight of train in lbs. $= 75 \times 2240 = 168000$ lbs.;

$$\text{Rise of rail per ft.} = \frac{1}{8} \div 100 = \frac{1}{800} \text{ ft.};$$

$$\therefore \quad " \quad " \quad 3520 \text{ ft.} = \frac{1}{800} \times 3520 = 4{\cdot}4 \text{ ft.};$$

$\therefore$ Work due to gravity per min. $= 4{\cdot}4 \times 168000 = 739200$;
" " friction per min. $= 6 \times 75 \times 3520 = 1584000$;
$\therefore$ Total work per min. $= 1584000 - 739200 = 844800$;

$$\text{And H. P.} = \frac{844800}{33000} = 25{\cdot}6.$$

Ex. 7.

$$\text{Speed of train per min.} = \frac{50 \times 5280}{60} = 4400 \text{ ft.}$$

Weight of train in lbs. $= 75 \times 2240 = 168000$ lbs.

$$\text{Rise of rail per ft.} = \frac{1}{8} \div 100 = \frac{1}{800} \text{ ft.};$$

$$\therefore \quad " \quad " \quad 4400 \text{ ft.} = \frac{1}{800} \times 4400 = 5{\cdot}5 \text{ ft.};$$

$\therefore$ Work due to gravity per min. $= 5{\cdot}5 \times 168000 = 924000$;
" " friction per min. $= 75 \times 7 \times 4400 = 2310000$.

Now as the train descends the gradient, we have
$\therefore$ Total work per min. $= 2310000 - 924000 = 1386000$;

$$\text{And H. P.} = \frac{1386000}{33000} = 42.$$

Ex. 8.

$$\text{Speed of train per min.} = \frac{50 \times 5280}{60} = 4400 \text{ ft.};$$

$$\text{Weight of train in lbs.} = 75 \times 2240 = 168000 \text{ lbs.};$$

$$\text{Rise of rail per ft.} = \frac{1}{4} \div 100 = \frac{1}{400};$$

$$\therefore \quad ,, \quad ,, \quad 4400 \text{ ft} = \frac{1}{400} \times 4400 = 11 \text{ ft.};$$

$$\therefore \text{ Work due to gravity per min.} = 11 \times 168000 = 1848000;$$

$$,, \quad ,, \quad \text{friction per min.} = 7 \times 75 \times 4400 = 2310000;$$

$$\therefore \text{ Total work per min} = 2310000 - 1848000 = 462000;$$

$$\text{And H. P.} = \frac{462000}{33000} = 14.$$

Ex. 9.

$$\text{Speed of train per min.} = \frac{50 \times 5280}{60} = 4400 \text{ ft.};$$

$$\text{Weight of train in lbs.} = 75 \times 2240 = 168000 \text{ lbs.};$$

$$\text{Rise of rail per ft.} = \frac{1}{4} \div 100 = \frac{1}{400} \text{ ft.};$$

$$\therefore \quad ,, \quad ,, \quad 4400 \text{ ft.} = \frac{1}{400} \times 4400 = 11 \text{ ft.};$$

$$\therefore \text{ Work due to gravity per min.} = 11 \times 168000 = 1848000;$$

$$,, \quad ,, \quad \text{friction per min.} = 7 \times 75 \times 4400 = 2310000;$$

$$\therefore \text{ Total work per min.} = 1848000 + 2310000 = 4158000;$$

$$\text{And H. P.} = \frac{4158000}{33000} = 126.$$

Ex. 10.

$$\text{Speed of train per min.} = \frac{50 \times 5280}{60} = 4400 \text{ ft.};$$

Weight of train in lbs. $= 60 \times 2240 = 134400$ lbs.;

$$\text{Rise of rail per ft.} = \frac{1}{8} \div 100 = \frac{1}{800} \text{ ft.};$$

$$\therefore \text{ „ „ } 4400 \text{ ft.} = \frac{1}{800} \times 4400 = 5{\cdot}5 \text{ ft.};$$

$\therefore$ Work due to gravity per min. $= 5{\cdot}5 \times 134400 = 739200$;

„ „ friction per min. $= 7 \times 60 \times 4400 = 1848000$;

$\therefore$ Total work per min. $= 1848000 - 739200 = 1108800$;

$$\text{And H. P.} = \frac{1108800}{33000} = 33{\cdot}6.$$

Ex. 11.

Resistance friction $= \frac{1}{12}$ of 24 cwts.

$= 2$ cwts. $= 2 \times 112$ lbs. $= 224$ lbs.;

Space in feet $= 5280$ ft.;

$\therefore$ Work due to friction $= 224 \times 5280 = 1182720$.

$$\text{Rise of rail per ft.} = \frac{2}{100} \text{ ft.};$$

$$\therefore \text{ „ „ } 5280 \text{ ft.} = \frac{2}{100} \times 5280 = 105{\cdot}6 \text{ ft.}$$

Wt. in lbs. $= 24 \times 112$;

$\therefore$ Work due to gravity $= 24 \times 112 \times 105{\cdot}6 = 283852$;

$\therefore$ Total work $= 283852 + 1182720 = 1466572$.

Ex. 12.

$$\text{Resistance of friction} = \tfrac{1}{12} \text{ of 16 cwts.} = \frac{448}{3} \text{ lbs.};$$

Space in feet $= 5280$ ft.;

$$\therefore \text{ Work due to friction} = \frac{448}{3} \times 5280 = 788480.$$

$$\text{Rise of rail per ft.} = \frac{2}{100} \text{ ft.};$$

$$\therefore \text{ „ „ } 5280 \text{ ft.} = \frac{2}{100} \times 5280 = 105{\cdot}6 \text{ ft.}$$

Wt. in lbs. $= 16 \times 112$ lbs.;

$\therefore$ Work due to gravity $= 16 \times 112 \times 105{\cdot}6 = 189235+$.

Total work $= 788480 + 189235 = 977715$.

ART. 13.

Ex. 1.

Number of relays $= \frac{720}{120} = 6$;

No. shovellers to do the work in 1 day $= \frac{50000}{500} = 100$;
" barrowmen " " " $= 100 \times 6 = 600$;
" pickmen " " " $= \frac{1}{3} \times 100 = 33\frac{1}{3}$;
Wages of shovellers $= 100 \times 3s. = 300s. = 15l.$;
" barrowmen $= 600 \times 3s. = 1800s. = 90l.$;
" pickmen $= 33\frac{1}{3} \times 3s.\ 4d. = 111s.\ 1\frac{1}{4}d.$ nearly
$= 5l.\ 11s.\ 1\frac{1}{4}d.$

$\therefore$ Total cost $= 15l. + 90l. + 5l.\ 11s.\ 1\frac{1}{4}d.$
$= 110l.\ 11s.\ 1\frac{1}{4}d.$

Ex. 2.

No. relays $= \frac{720}{120} = 6$;

No. shovellers to do work in one day $= \frac{30000}{500} = 60$;
" barrowmen " " " $= 60 \times 6 = 360$;
" pickmen " " " $= \frac{1}{3} \times 60 = 20$;
Wages of shovellers $= 60 \times 3s. = 180s. = 9l.$;
" barrowmen $= 360 \times 3s. = 1080s. = 54l.$;
" pickmen $= 20 \times 3s.\ 4d. = 3l.\ 6s.\ 8d.$;

$\therefore$ Total cost $= 9l. + 54l. + 3l.\ 6s.\ 8d.$
$= 66l.\ 6s.\ 8d.$

Ex. 3.

No. planks in the ramp $= 8$;

$\therefore$ Length of ramp $= 12 \times 8 = 96$ ft.

relays on the ramp $=\frac{96}{80}=1{\cdot}2$;

relays on the level $=\frac{720}{120}=6$;

∴ Total no. relays $=1{\cdot}2+6=7{\cdot}2$.

ys' work of shovellers $=\frac{50000}{500}=100$;

" " barrowmen $=100\times7{\cdot}2=720$;

" " pickmen $=100\times\frac{1}{3}=33\frac{1}{3}$;

Vages of shovellers $=100\times3s.=300s.=15l.$;

" barrowmen $=720\times3s.=2160s.=108l.$;

" pickmen $=33\frac{1}{3}\times3s.\ 4d.=111s.\ 1\frac{1}{4}d.$ nearly;

∴ Total cost $=128l.\ 11s.\ 1\frac{1}{4}d.$ nearly.

Ex. 4.

No. relays on level $=\frac{480}{120}=4$;

Content of excavation $=200\times40\times25=200000$ c. ft.;

∴ Days' work of shovellers $=\frac{200000}{500}=400$;

" " barrowmen $=4\times400=1600$;

" " pickmen $=\frac{4\times400}{3}=533\frac{1}{3}$;

∴ Total no. of days in completing the excavation = 2533

∴ Cost $=2533\frac{1}{3}\times2s.\ 6d.=316l.\ 13s.\ 4d.$

Ex. 5.

No. relays on level $=\frac{480}{120}=4$;

Days' work of shovellers $=\frac{80000}{500}=160$;

" " barrowmen $=4\times160=640$;

" " pickmen $=\frac{4\times160}{3}=213\frac{1}{3}$;

∴ Total no. of days in completing the excavation $= 1013\frac{1}{3}$;

And cost $= 2s.\ 6d. \times 1013\frac{1}{3} = 126l.\ 13s.\ 4d.$

Ex. 6.

No. relays $= \frac{380}{120} = 3\frac{1}{6}$;

Days' work of shovellers $= \frac{200000}{500} = 400$;

" " barrowmen $= 3\frac{1}{6} \times 400 = 1266\frac{2}{3}$;

" " pickmen $= \frac{3 \times 400}{2} = 600$;

∴ Total no. of days $= 2266\frac{2}{3}$;

And cost $= 2s.\ 6d. \times 2266\frac{2}{3} = 283l.\ 6s.\ 8d.$

Ex. 7.

No. relays $= \frac{380}{120} = 3\frac{1}{6}$;

Days' work of shovellers $= \frac{5000}{500} = 10$;

" " barrowmen $= 3\frac{1}{6} \times 10 = 31\frac{2}{3}$;

" " pickmen $= \frac{3 \times 10}{2} = 15$;

∴ Total no. days $= 10 + 31\frac{2}{3} + 15 = 56\frac{2}{3}$;

∴ Cost $= 2s.\ 6d. \times 56\frac{2}{3}$

$= 7l.\ 1s.\ 8d.$

Ex. 8.

Here, by Ex. 4., we have

No. shovellers to complete the excavation in 1 day

$= 400$;

∴ " " " " in 40 days

$= \frac{1}{40}$ of 400

$= 10$ shovellers;

$\therefore$ No. barrowmen $= 10 \times 4 = 40$;

$\therefore$ No. pickmen $= \frac{4 \times 10}{3} = 13\frac{1}{3}$.

Ex. 9.

No. relays $= \frac{480}{120} = 4$;

Content of excavation $= 140 \times 40 \times 25 = 140000$ c. ft.;

Days' work of shovellers $= \frac{140000}{500} = 280$;

„ „ barrowmen $= 280 \times 4 = 1120$;

„ „ pickmen $= \frac{4 \times 280}{3} = 373\frac{1}{3}$;

$\therefore$ Total no. days $= 1773\frac{1}{3}$ days;

And cost $= 2s.\ 6d. \times 1773\frac{1}{3}$

$= 221l.\ 13s.\ 4d.$

Ex. 10.

No. planks on the ramp $= 13$;

$\therefore$ Length of ramp $= 12 \times 13 = 156$ ft.;

No. of relays on the ramp $= \frac{156}{80} = 1{\cdot}95$;

No. of relays on the level $= \frac{480}{120} = 4$;

$\therefore$ Total no. of relays $= 1{\cdot}95 + 4 = 5{\cdot}95$.

Days' work of shovellers $= \frac{200000}{500} = 400$;

„ „ barrowmen $= 5{\cdot}95 \times 400 = 2380$;

„ „ pickmen $= \frac{4 \times 400}{3} = 533\frac{1}{3}$;

$\therefore$ Total no. of days $= 3313\frac{1}{3}$ days,

And cost $= 2s.\ 6d. \times 3313\frac{1}{3}$

$= 414l.\ 3s.\ 4d.$

Ex. 11.

No. of planks on the ramp $= 10$;

$\therefore$ Length of ramp $= 12 \times 10 = 120$ ft.

No. relays on the ramp $= \frac{120}{80} = 1{\cdot}5$;

Days' work of shovellers $= \frac{90000}{500} = 180$;

" " barrowmen $= 1{\cdot}5 \times 180 = 270$;

" " pickmen $= \frac{3}{5} \times 180 = 108$;

$\therefore$ Total no. of days $= 558$.

Now, since the work is to be done in 20 days, the number of men will be the $\frac{1}{20}$th of the number required to do the work in 1 day.

$\therefore$ No. of men to do the work in 20 days $= \frac{558}{20} = 27{\cdot}9$.

Ex. 12.

No. of planks on the ramp $= 10$;

Length of ramp $= 10 \times 12 = 120$;

No. of relays on ramp $= \frac{120}{80} = 1{\cdot}5$;

Days' work of shovellers $= \frac{60000}{500} = 120$;

" " barrowmen $= 120 \times 1{\cdot}5 = 180$;

" " pickmen $= \frac{3}{5} \times 120 = 72$;

$\therefore$ Total no. of days $= 372$,

And no. of men to do work in 20 days $= \frac{372}{20} = 18{\cdot}6$.

ART. 14.

Ex. 1.

Speed per min. $= \frac{40 \times 5280}{60} = 3520$;

Resistance of air $=\left(\frac{40}{10}\right)^2\times 50=800$ lbs.;

Work due to resistance of air $=800\times 3520=2816000$;

$$\therefore\ \text{H. P.}=\frac{2816000}{33000}=85{\cdot}3.$$

Ex. 2.

Speed per min. $=\frac{20\times 5280}{60}=1760$;

Resistance of air $=\left(\frac{20}{10}\right)^2\times 50=200$ lbs.;

Work due to resistance of air $=200\times 1760=352000$;

$$\therefore\ \text{H. P.}=\frac{352000}{33000}=10{\cdot}6.$$

Ex. 3.

Speed per min. $=\frac{20\times 5280}{60}=1760$;

Resistance of air $=\left(\frac{20}{10}\right)^2\times 15=60$ lbs.;

Work due to resistance of air $=60\times 1760=105600$;

Work due to friction $=6\times 20\times 1760=213200$;

$\therefore$ Total work per min. $=105600+213200=318800$,

$$\text{And H. P.}=\frac{318800}{33000}=9{\cdot}6.$$

Ex. 4.

Speed per min. $=\frac{30\times 5280}{60}=2640$ ft.;

Resistance of air $=\left(\frac{30}{10}\right)^2\times 15=135$ lbs.;

$\therefore$ Work due to the resistance of the air per min.

$=135\times 2640=356400$.

Work due to friction per min. $= 6 \times 20 \times 2640$
$= 316800$;

$\therefore$ Total work per min. $= 356400 + 316800$
$= 673200$;

$$\therefore \text{H. P.} = \frac{673200}{33000} = 20{\cdot}4.$$

Ex. 5.

Speed per min. $= \dfrac{40 \times 5280}{60} = 3520$ ft.;

Resistance air $= \left(\dfrac{40}{10}\right)^2 \times 40 = 640$ lbs.;

$\therefore$ Work due to the resistance of the air per min.
$= 640 \times 3520 = 2252800$.

Work due to friction per min. $= 6\frac{1}{2} \times 90 \times 3520$
$= 2059200$;

$\therefore$ Total work per min. $= 2252800 + 2059200$
$= 4312000$;

$$\therefore \text{H. P.} = \frac{4312000}{33000} = 130{\cdot}6.$$

Ex. 6.

Speed per min. $= \dfrac{50 \times 5280}{60} = 4400$;

Resistance air $= \left(\dfrac{50}{10}\right)^2 \times 40 = 1000$;

$\therefore$ Work due to the resistance of the air per min.
$= 1000 \times 4400 = 4400000$.

Work due to friction per min. $= 6\frac{1}{2} \times 90 \times 4400$
$= 2574000$;

$\therefore$ Total work per min. $= 4400000 + 2574000$
$= 6974000$;

$$\therefore \text{H. P.} = \frac{6974000}{33000} = 211.$$

Ex. 7.

Speed per min. $= \dfrac{60 \times 5280}{60} = 5280$ ft.;

Resistance air $= \left(\dfrac{60}{10}\right)^2 \times 35 = 1260$ lbs.;

$\therefore$ Work due to the resistance of the air per min.
$= 1260 \times 5280 = 6652800$;

Work due to friction per min. $= 6 \times 60 \times 5280$
$= 1900800$;

$\therefore$ Total work per min. $= 6652800 + 1900800$
$= 8553600$;

$$\therefore \text{ H.P.} = \frac{8553600}{33000} = 259.$$

Ex. 8.

Speed per min. $= \dfrac{30 \times 5280}{60} = 2640$ ft.;

Total resistance of air $= 400$ lbs.;

$\therefore$ Work due to the resistance of the air per min.
$= 400 \times 2640 = 1056000$;

Work due to friction per min. $= 6 \times 70 \times 2640$
$= 1108800$;

$\therefore$ Total work per min. $= 1056000 + 1108800$
$= 2164800$;

$$\therefore \text{ H.P.} = \frac{2164800}{33000} = 65{\cdot}6.$$

ART. 15.

Ex. 1.

Work of engine per hour $= 33000 \times 8 \times 60$
$= 15840000$.

Effective work of engine per hour $= 15840000 \times \frac{2}{3}$
$= 10560000$.

Depth of the pit in ft. $= 160 \times 6 = 960$ ft.;

$\therefore$ Work in raising one cubic ft. of water $= 960 \times 62{\cdot}5$
$= 60000$;

$\therefore$ No. cubic ft. raised per hour $= \frac{10560000}{60000} = 176$.

Or thus.

Let $x =$ the no. of cubic feet of water raised per hour.
Wt. water in lbs. $= 62{\cdot}5 \times x$.
Depth of the pit in ft. $= 960$ ft;
$\therefore$ Work in raising water per hour $= 62{\cdot}5 \times x \times 960$.
But this is equal to the effective work of the engine per hour;

$$\therefore\ 62{\cdot}5 \times x \times 960 = 10560000;$$

$$\therefore\ x = \frac{10560000}{62{\cdot}5 \times 960} = 176 \text{ c. ft.}$$

Ex. 2.

Work of engine per hour $= 33000 \times 8 \times 60$
$= 15840000$.
Effective work engine per hour $= 15840000 \times \frac{2}{3}$
$= 10560000$.
Depth of the pit in ft. $= 200 \times 6 = 1200$ ft.;
$\therefore$ Work in raising one cubic foot of water $= 1200 \times 62{\cdot}5$
$= 75000$;

$\therefore$ No. cubic ft. raised per hour $= \frac{10560000}{75000} = 140{\cdot}8$.

Ex. 3.

Wt. water raised per min. $= 2{\cdot}2 \times 62{\cdot}5$ lbs. $= 137{\cdot}5$ lbs.
Depth of the pit in ft. $= 160 \times 6 = 960$ ft.
Work in raising $2{\cdot}2$ cubic ft. of water, or work per min.
$= 137{\cdot}5 \times 960 = 132000$.

Effective work of 1 H. P. of the engine per min.

$$= 33000 \times \tfrac{2}{3}$$
$$= 22000;$$

$$\therefore \text{ H. P.} = \frac{132000}{22000} = 6.$$

Or thus.

Let $x =$ the no. of horse powers.

Effective work engine per min. $= 33000 \times x \times \tfrac{2}{3}$.

Work in raising the water per min. $= 2{\cdot}2 \times 62{\cdot}5 \times 160 \times 6$

$$= 132000;$$

$$\therefore \ 33000 \times x \times \tfrac{2}{3} = 132000,$$

$$x = \frac{132000 \times 3}{33000 \times 2} = 6.$$

Ex. 4.

Work in raising 1·65 c. ft. of water $= 1{\cdot}65 \times 62\ 5 \times 160 \times 6$

$$= 99000.$$

Effective work of engine per min. $= 33000 \times \tfrac{2}{3}$

$$= 22000;$$

$$\therefore \text{ H. P.} = \frac{99000}{22000} = 4{\cdot}5.$$

Ex. 5.

Wt. water $= 44 \times 62{\cdot}5 = 2750$ lbs.

Depth in ft. $= 160 \times 6 = 960$ ft.

Work in raising 44 c. ft. of water $= 2750 \times 960 = 2640000$.

Effective work of engine per min. $= 8 \times 33000 \times \tfrac{2}{3}$

$$= 176000;$$

$$\therefore \text{ No. of minutes} = \frac{2640000}{176000} = 15.$$

Or thus.

Let $x =$ the no. minutes required to pump the water.

Effective work engine $= 8 \times 33000 \times \frac{2}{3} \times x$
$= 176000 \times x$.

But this is equal to the work in raising the water;

$$\therefore\ 176000 \times x = 2640000;$$

$$\therefore\ x = \frac{2640000}{176000} = 15.$$

Ex. 6.

Work required to be done per day $= 800 \times 20 \times 62 \cdot 5 \times 60$
$= 60000000$.

Effective work of 1 H. P. of the engine per day $= 33000 \times 60 \times 8 \times \frac{2}{3}$
$= 10560000$;

$$\therefore\ \text{H. P.} = \frac{60000000}{10560000} = 5 \cdot 68.$$

Ex. 7.

Work required to be done per day $= 800 \times 9 \times 62 \cdot 5 \times 60$
$= 27000000$.

Effective work of 1 H. P. per day $= 33000 \times 60 \times 8 \times \frac{2}{3}$
$= 10560000$;

$$\therefore\ \text{H. P.} = \frac{27000000}{10560000} = 2 \cdot 6 \text{ nearly.}$$

ART. 17.

Ex. 1.

Weight of water $= 200 \times 62 \cdot 5 = 12500$ lbs.
Height of the fall $= 20$ ft.;
$\therefore$ Work of water per min. $= 12500 \times 20 = 250000$.
Modulus of the wheel $= \frac{2}{3}$;
$\therefore$ Effective work of the wheel per min. $= 250000 \times \frac{2}{3}$
$= 166666\frac{2}{3}$;

$$\therefore \text{ H. P.} = \frac{166666\frac{2}{3}}{33000} = 5+.$$

Ex. 2.

Weight of water $= 80 \times 62 \cdot 5 = 5000$.

Height of the fall $= 20$ ft.;

$\therefore$ Work of water per min. $= 5000 \times 20 = 100000$.

Effective work of wheel per min. $= 100000 \times \frac{2}{3}$

$= 66666\frac{2}{3}$;

$$\therefore \text{ H. P.} = \frac{66666\frac{2}{3}}{33000} = 2+.$$

Ex. 3.

Water going over the fall per min. $= 2 \times 1 \times 50$

$= 100$ cubic feet;

$\therefore$ Weight of water $= 100 \times 62 \cdot 5 = 6250$ lbs.

Height of the fall $= 16$ ft;

$\therefore$ Work of water per min. $= 6250 \times 16 = 100000$.

As the water wheel in this case is supposed to take up all the work, we have

$$\therefore \text{ H. P.} = \frac{100000}{33000} = 3 \cdot 03.$$

Ex. 4.

Water going over the fall per min. $= 2 \times 1 \times 40$

$= 80$ cubic feet;

$\therefore$ Weight of water $= 80 \times 62 \cdot 5 = 5000$ lbs.

Height of the fall $= 16$ ft.;

$\therefore$ Work of water per min. $= 5000 \times 16 = 80000$;

$$\therefore \text{ H. P.} = \frac{80000}{33000} = 2 \cdot 4.$$

Ex. 5.

Water going over the fall per min. $= 2 \times 1 \times 50$
$= 100$ cubic feet;

$\therefore$ Weight of water $= 100 \times 62{\cdot}5 = 6250$ lbs.;

$\therefore$ Work of water per min. $= 6250 \times 16 = 100000$.

Effective work of the wheel per min. $= 100000 \times {\cdot}6$
$= 60000$;

$$\therefore \text{ H. P.} = \frac{60000}{33000} = 1{\cdot}8.$$

Ex. 6.

Water going over the fall per min. $= 3 \times 2 \times 40$
$= 240$ cubic feet;

$\therefore$ Weight of water $= 240 \times 62{\cdot}5 = 15000$ lbs.

Height of the fall $= 10$ ft.;

$\therefore$ Work of water per min. $= 15000 \times 10 = 150000$;

$\therefore$ Effective work of wheel per min. $= 150000 \times \frac{6}{10}$
$= 90000$;

$$\therefore \text{ H. P.} = \frac{90000}{33000} = 2{\cdot}7.$$

Ex. 7.

Water going over the fall per min. $= 3 \times 2 \times 40$
$= 240$ cubic feet;

$\therefore$ Weight of water $= 240 \times 62{\cdot}5 = 15000$ lbs.

Height of the fall $= 8$ ft.

Work of water per min. $= 15000 \times 8 = 120000$.

Effective work of the wheel per min. $= 120000 \times \frac{6}{10}$
$= 72000$;

$$\therefore \text{ H. P.} = \frac{72000}{33000} = 2{\cdot}1.$$

Ex. 8.

Water going over the fall per second $= 4 \times 3 \times 1{\cdot}5$ c. ft.;

$\therefore$ Water going over the fall per min. $= 4 \times 3 \times 1{\cdot}5 \times 60$
$= 1080$ cubic feet;

$\therefore$ Weight of water $= 1080 \times 62{\cdot}5 = 67500$ lbs.

Height of the fall $= 9$ ft.;

$\therefore$ Work of water $= 67500 \times 9 = 607500$.

Effective work of wheel per min. $= 607500 \times {\cdot}68$
$= 413100$;

$$\therefore \text{ H. P.} = \frac{413100}{33000} = 12{\cdot}5.$$

Ex. 9.

Water going over the fall per min. $= 4 \times 3 \times {\cdot}5 \times 60$
$= 360$ cubic feet;

$\therefore$ Weight of water $= 360 \times 62{\cdot}5 = 22500$ lbs.

Height of the fall $= 9$ ft.;

$\therefore$ Work of water $= 22500 \times 9 = 202500$.

Effective work of the wheel per min. $= 202500 \times {\cdot}68$
$= 137700$;

$$\therefore \text{ H. P.} = \frac{137700}{33000} = 4{\cdot}1.$$

ARTS. 22. AND 23.

Ex. 1.

Work done upon one inch of the piston in 1 stroke
lbs. pressure steam per sq. in. × length stroke $= 30 \times 8 = 240$;

$\therefore$ Work done upon the whole piston in 1 stroke
work done on 1 sq. in. of piston × no. sq. inches in the piston
$240 \times 500 = 120000$;

∴ Work done upon the whole piston in 20 strokes

$$=120000\times 20=2400000.$$

As this is the work done per min., we have

$$\text{H. P.}=\frac{2400000}{33000}=72{\cdot}7.$$

Ex. 2.

Work done upon one inch of the piston in one stroke

$$=20\times 8=160;$$

∴ Work done upon the whole piston in one stroke

$$=160\times 500=80000;$$

∴ Work done upon the whole piston in 20 strokes

$$=20\times 80000=1600000;$$

$$\therefore\ \text{H. P.}=\frac{1600000}{33000}=48{\cdot}4.$$

Ex. 3.

Work of engine per min. $=24\times 33000=792000$.

Work of 1 lb. pressure of steam in 1 stroke $=1\times 500\times 8$;

∴ Work of 1 lb. pressure of steam per min. $=500\times 8\times 20$

$$=80000;$$

$$\therefore\ \text{No. lbs. pressure}=\frac{792000}{80000}=10\text{ lbs. nearly.}$$

Or thus.

Let $x=$ the mean effective pressure of the steam in lbs. upon each square inch of the piston.

Work of steam on the whole piston in 1 stroke

$$=x\times 8\times 500;$$

∴ Work of steam on the whole piston per min.

$$=x\times 8\times 500\times 20.$$

But this must be equal to the work of the engine per minute;

$$\therefore\ x\times 8\times 500\times 20=792000;$$

$$\therefore\ x = \frac{792000}{8 \times 500 \times 20} = 10 \text{ lbs. nearly.}$$

Ex. 4.

Work done upon one inch of the piston in 1 stroke

$$= 9 \times 7{\cdot}9 = 71{\cdot}1;$$

$\therefore$ Work done upon the whole piston in one stroke

$$= 71{\cdot}1 \times 2800$$
$$= 199080;$$

$\therefore$ Work done upon the whole piston in 11·5 strokes, or per min. $= 199080 \times 11{\cdot}5$

$$= 2289420;$$

$$\therefore\ \text{H. P.} = \frac{2289420}{33000} = 69{\cdot}3.$$

Ex. 5.

Work done upon one inch of the piston in 1 stroke

$$= 9 \times 7{\cdot}9 = 71{\cdot}1;$$

$\therefore$ Work done upon the whole piston in one stroke

$$= 71{\cdot}1 \times 700 = 49770;$$

$\therefore$ Work done upon the whole piston in 23 strokes

$$= 23 \times 49770 = 1144710;$$

$$\therefore\ \text{H. P.} = \frac{1144710}{33000} = 34{\cdot}6.$$

Ex. 6.

Volume water in the cylinder of pump $= 9 \times 10$ c. ft.

Work in raising water at each double stroke

$$= 9 \times 10 \times 62{\cdot}5 \times 108 = 607500;$$

$\therefore$ Work in raising water per min. $= 607500 \times 8$.

Work steam per min. at 1 lb. pressure on each inch of the piston $= 5000 \times 10 \times 8$;

$$\therefore\ \text{No. lbs.} = \frac{607500 \times 8}{5000 \times 10 \times 8} = 12{\cdot}15 \text{ lbs.}$$

Or thus.

Let $x =$ no. lbs. effective pressure on each inch of the piston; then

$$\text{No. lbs. pressure on piston} = x \times 5000;$$

Work steam per min. $= x \times 5000 \times 10 \times 8$;

And, as in the foregoing solution,

$$\text{Work in raising water per min.} = 607500 \times 8.$$

But the work of the steam is expended in raising the water;

$$\therefore\ x \times 5000 \times 10 \times 8 = 607500 \times 8;$$

$$\therefore\ x = \frac{607500 \times 8}{5000 \times 10 \times 8} = 12{\cdot}15 \text{ lbs.}$$

Ex. 7.

Here the effective work per min. $= 607500 \times 8$;

$$\therefore\ \text{Useful H. P.} = \frac{607500 \times 8}{33000} = 147{\cdot}2.$$

Ex. 8.

Effective work of engine per min. $= 20 \times 33000$
$= 660000$;

Work in overcoming friction per min. $= 2 \times 300 \times 10 \times 12$
$= 72000$;

$\therefore$ Total work of engine per min. $=$ effective work $+$ work friction $= 660000 + 72000 = 732000$.

Work of steam per min. at 1 lb. pressure on each inch of piston $= 1 \times 300 \times 10 \times 12 = 36000$;

$$\therefore\ \text{No. lbs. pressure of steam per min.} = \frac{732000}{36000} = 20\tfrac{1}{3} \text{ lbs.}$$

Or thus.

Let $x =$ the no. lbs. pressure of the steam.

Work of steam in each stroke $= x \times 300 \times 10$;

$\therefore$ Work of steam in 12 strokes, or per min.

$$= x \times 300 \times 10 \times 12;$$

This must be equal to the total work of the engine per min.;

$$\therefore\ x \times 300 \times 10 \times 12 = 732000;$$

$$\therefore\ x = \frac{732000}{300 \times 10 \times 12} = 20\tfrac{1}{3} \text{ lbs.}$$

Ex. 9.

Here, as in the last Example,

Effective work of engine per min. $= 15 \times 33000$

$= 495000;$

Work in overcoming friction per min. $= 2 \times 300 \times 5 \times 12$

$= 36000;$

$\therefore$ Total work of engine per min. $= 495000 + 36000$

$= 531000.$

Work of steam per min. at 1 lb. pressure on each inch of piston $= 1 \times 300 \times 5 \times 12 = 18000;$

$$\therefore \text{ No. lbs. pressure of steam per in.} = \frac{531000}{18000}$$

$$= 29{\cdot}5.$$

Ex. 10.

Work steam in 1 stroke $= 45 \times 400 \times 3;$

$\therefore$ Work steam per min. $= 45 \times 400 \times 3 \times 18$

$= 972000;$

Work due to the resistance of friction and the pressure of the atmosphere per min. $= 18 \times 400 \times 3 \times 18$

$= 388800;$

$\therefore$ Effective work steam per min. $= 972000 - 388800$

$= 583200;$

$$\therefore \text{ Useful H. P.} = \frac{583200}{33000} = 17{\cdot}67.$$

Volume cylinder, or steam discharged each stroke $= \frac{400}{144} \times 3;$

$$\text{Volume steam discharged per min.} = \frac{400}{144} \times 3 \times 18$$
$$= 150 \text{ c. ft.}$$

Now, by the table, at 45 lbs. pressure, 610 c. ft. of steam is raised from 1 c. ft. of water;

$$\therefore \text{ No. c. ft. of water evaporated per min.} = \frac{150}{610}$$
$$= \cdot 246 \text{ c. ft. nearly.}$$

Ex. 11.

Work of steam in 1 stroke $= 48 \times 400 \times 3$;

$$\therefore \text{ Work of steam per min.} = 48 \times 400 \times 3 \times 18$$
$$= 1036800.$$

As in the last Example,

Work due to the resistance of friction and pressure of atmosphere per min. $= 18 \times 400 \times 3 \times 18 = 388800$;

$$\text{Effective work steam per min.} = 1036800 - 388800$$
$$= 648000;$$

$$\therefore \text{ Useful H.P.} = \frac{648000}{33000} = 19\cdot 6.$$

Volume cylinder, or steam discharged each stroke, $= \frac{400}{144} \times 3$;

$$\text{Volume steam discharged per min.} = \frac{400}{144} \times 3 \times 18$$
$$= 150 \text{ c. ft.}$$

Now, by the table, at 48 lbs. pressure, 575 c. ft. of steam is raised from 1 c. ft. of water;

$$\therefore \text{ No. c. ft. of water evaporated per min.} = \frac{150}{575} = \cdot 26 \text{ c. ft.}$$

Ex. 12.

by the solution to Ex. 10., we have
done by ·246 c. ft. water $= 583200$;

ork done by 11·5 c. ft. water $= \dfrac{583200 \times 11{\cdot}5}{\cdot 246}$

$$= 27 \text{ millions} + ;$$

the work done by a bushel of coals, or the duty of
ne.

Ex. 13.

by the solution to Ex. 11., we have,
done by ·26 c. ft. water $= 648000$;

done by 11·5 c. ft. water $= \dfrac{648000 \times 11{\cdot}5}{\cdot 26}$

$$= 28 \text{ millions} + ;$$

s the duty of the engine, or the work done by a
f coals.

ART. 28.

Ex. 1.

the arm by which P acts = 2 ft. = 24 in.; the arm
h W acts = 3 in.; and W = 64 lbs. to find P.

$$\therefore P \times 24 = 64 \times 3;$$

$$\therefore P = \frac{64 \times 3}{24} = \frac{64}{8} = 8 \text{ lbs.}$$

Ex. 2.

$$P \times 24 = 16 \times 3;$$

$$\therefore P = \frac{16 \times 3}{24} = \frac{16}{8} = 2 \text{ lbs.}$$

Ex. 3.

the arm by which P acts = 24 in.; P = 14 lbs.; and
by which W acts = 3 in., to find W.

$$\therefore \text{W} \times 3 = 14 \times 24;$$

$$\therefore \text{W} = \frac{14 \times 24}{3} = 14 \times 8 = 112 \text{ lbs.}$$

$$= 1 \text{ cwt.}$$

Ex. 4.

Let x = the distance of the power, in inches, from the fulcrum; then

$$\text{Moment of the power} = 7 \times x;$$

$$\text{,, \quad ,, \quad weight} = 126 \times 2;$$

$$\therefore 7 \times x = 126 \times 2,$$

$$\therefore x = \frac{126 \times 2}{7} = 18 \times 2 = 36 \text{ inches}$$

$$= 3 \text{ feet.}$$

Ex. 5.

In this case we have

$$6 \times x = 126 \times 2;$$

$$\therefore x = \frac{126 \times 2}{6} = \frac{126}{3} = 42 \text{ inches.}$$

Ex. 6.

Here a power of 60 lbs. is applied at the distance of 30 in. from the fulcrum; the weight, W, or the resistance, acts at the distance of 3 in. from the fulcrum; and W is required;

$$\therefore \text{Moment of the power} = 60 \times 30;$$

$$\text{,, \quad ,, \quad weight} = \text{W} \times 3;$$

$$\therefore \text{W} \times 3 = 60 \times 30;$$

$$\therefore \text{W} = \frac{60 \times 30}{3} = 60 \times 10 = 600 \text{ lbs.}$$

Ex. 7.

In this case we have,

$$W \times 3 = 47 \times 30;$$

$$\therefore\ W = \frac{47 \times 30}{3} = 47 \times 10 = 470 \text{ lbs.}$$

Ex. 8.

Here a power of 2 tons is applied at the distance of 10 ft. from the fulcrum; the weight, W, or the resistance, acts at the distance of 9 ft. from the fulcrum; and W is required;

$$\therefore\ \text{Moment of the power} = 2 \times 10;$$

$$\text{,, \quad ,, \quad weight} = W \times 9;$$

$$\therefore\ W \times 9 = 2 \times 10;$$

$$\therefore\ W = \frac{2 \times 10}{9} = \frac{20}{9} = 2\tfrac{2}{9} \text{ tons.}$$

Ex. 9.

In this case we have,

$$W \times 8 = 2 \times 10;$$

$$\therefore\ W = \frac{2 \times 10}{8} = \frac{20}{8} = 2\tfrac{1}{2} \text{ tons.}$$

Ex. 10.

Total pressure of the steam on the safety valve
$= 50$ lbs. $\times 4 = 200$ lbs.;

Effective pressure of the steam upon the lever
= Total pressure steam − wt. valve
$= 200$ lbs. $- 6$ lbs. $= 194$ lbs.

Now this pressure is applied at the distance of 2 inches from the fulcrum F;

$\therefore$ Moment of the pressure of the steam $= 194 \times 2 = 388$.

Moment of the load $W = 20 \times$ length lever;

$$\therefore\ 20 \times \text{length lever} = 388;$$

$$\therefore\ \text{Length lever} = \frac{388}{20} = 19{\cdot}4 \text{ inches.}$$

Ex. 11.

In this case we have,

Total pressure of the steam on the safety valve

$$= 30 \text{ lbs.} \times 4 = 120 \text{ lbs};$$

Effective pressure of the steam upon the lever

$$= 120 \text{ lbs.} - 6 \text{ lbs.} = 114 \text{ lbs.};$$

Moment of the pressure of the steam $= 114 \times 2 = 228$;

$$\therefore\ 20 \times \text{length lever} = 224;$$

$$\therefore\ \text{Length lever} = \frac{224}{20} = 11{\cdot}2 \text{ inches.}$$

ART. 38.

Ex. 1.

Here a power of 70 lbs. is applied at the distance of 20 inches from the centre of motion; a weight, W, acts at the distance of 4 inches from the centre of motion; and W is required;

$$\therefore\ \text{Moment of the power} = 70 \times 20 = 1400;$$

$$\text{,, ,, weight} = W \times 4;$$

$$\therefore\ W \times 4 = 1400,$$

$$\therefore\ W = \frac{1400}{4} = 350 \text{ lbs.};$$

Ex. 2.

In this case we have,

$$W \times 3 = 70 \times 20,$$

$$\therefore\ W = \frac{70 \times 20}{3} = \frac{1400}{3} = 466\tfrac{2}{3} \text{ lbs.}$$

Ex. 3.

Here a weight of 400 lbs. acts at the distance of 4 inches from the centre of motion; a power, P, is applied at the distance of 20 inches; and P is required when equilibrium takes place;

$$\therefore \text{ Moment of } W = 400 \times 4 \cdot$$
$$\text{,, } \quad \text{,, } \quad P = P \times 20;$$
$$\therefore P \times 20 = 400 \times 4,$$
$$\therefore P = \frac{400 \times 4}{20} = 20 \times 4 = 80 \text{ lbs.}$$

Ex. 4.

Let x = the radius of the axle in inches; then,

$$\text{Moment of } W = 4 \times 112 \times x;$$
$$\text{,, } \quad \text{,, } \quad P = 60 \times 2 \times 12;$$
$$\therefore 4 \times 112 \times x = 60 \times 2 \times 12;$$
$$\therefore x = \frac{60 \times 2 \times 12}{4 \times 112} = 3\cdot2 + \text{ inches.}$$

Ex. 5.

In this case we have,

$$4 \times 112 \times x = 60 \times 20;$$
$$\therefore x = \frac{60 \times 20}{4 \times 112} = 2\cdot6 + \text{ inches.}$$

Art. 39.

Ex. 1.

First, on the principle of work. Let the handle H (see *fig.* 73. of the text) make one revolution; then we have,

$$\text{Space moved over by } P = 2 \times 2 \times 3\cdot1416 \text{ ft.};$$
$$\therefore \text{ Work of } P = 4 \times 3\cdot1416 \times P.$$
$$\text{Circumference large axle} = \tfrac{1}{2} \times 3\cdot1416;$$
$$\text{,, } \quad \text{small axle} = \tfrac{1}{8} \times 3\cdot1416;$$

$\therefore$ Space moved over by W in ft. $= \frac{1}{2}$ of $\{\frac{1}{2} \times 3\cdot1416 - \frac{1}{8} \times 3\cdot1416\} = 3\cdot1416 \times \frac{1}{2}(\frac{1}{2} - \frac{1}{8}) = 3\cdot1416 \times \frac{1}{2} \times \frac{3}{8} = 3\cdot1416 \times \frac{3}{16}$;

$$\therefore \text{ Work of } W = 800 \times 3\cdot1416 \times \tfrac{3}{16}.$$

Now as the work of P is equal to the work of W friction being neglected, we have,

$$4\times 3{\cdot}1416\times P=800\times 3{\cdot}1416\times\tfrac{3}{16};$$

$$\therefore\ 4\times P=800\times\tfrac{3}{16};$$

$$\therefore\ P=\frac{800\times 3}{4\times 16}=\frac{100\times 3}{4\times 2}=\frac{300}{8}$$

$$=37{\cdot}5\text{ lbs.}$$

Second, on the principle of the equality of moments.

Radius large axle $=\frac{1}{2}$ of $\frac{1}{2}$ ft. $=\frac{1}{4}$ ft.;

„ small axle $=\frac{1}{2}$ of $\frac{1}{8}$ ft. $=\frac{1}{16}$ ft.

Now each cord sustains $\frac{1}{2}$ of 800 lbs. = 400 lbs.;

$\therefore$ Moment force applied to the large axle $=400\times\frac{1}{4}=100$;

„ „ „ „ small axle $=400\times\frac{1}{16}=25$;

Moment force applied to the handle $=2\times P$.

But the sum of the two latter moments must be equal to the former;

$$\therefore\ 2\times P+25=100;$$

$$\therefore\ 2\times P=75;$$

$$\therefore\ P=\frac{75}{2}=37{\cdot}5\text{ lbs.}$$

Ex. 2.

First, on the principle of work. As in the last example, we have

$$\text{Work of } P=4\times 3{\cdot}1416\times P.$$

Circumference large axle $=\frac{1}{2}\times 3{\cdot}1416$;

„ small axle $=\frac{3}{8}\times 3{\cdot}1416$;

$\therefore$ Space moved over by W in ft. $=\frac{1}{2}$ of $\{\frac{1}{2}\times 3{\cdot}1416-\frac{3}{8}\times 3{\cdot}1416\}=3{\cdot}1416\times\frac{1}{2}(\frac{1}{2}-\frac{3}{8})=3{\cdot}1416\times\frac{1}{2}\times\frac{1}{8}=3{\cdot}1416\times\frac{1}{16}$;

$$\therefore\ \text{Work of } W=800\times 3{\cdot}1416\times\tfrac{1}{16};$$

$$\therefore\ 4\times 3{\cdot}1416\times P=800\times 3{\cdot}1416\times\tfrac{1}{16};$$

$$\therefore\ 4P=800\times\tfrac{1}{16};$$

$$\therefore\ P=\frac{800}{4\times 16}=\frac{100}{8}=12{\cdot}5\text{ lbs.}$$

Second, on the principle of the equality of moments.

$$\text{Radius large axle} = \tfrac{1}{2} \text{ of } \tfrac{1}{2} \text{ ft.} = \tfrac{1}{4} \text{ ft.};$$

$$\text{,, small axle} = \tfrac{1}{2} \text{ of } \tfrac{3}{8} \text{ ft.} = \tfrac{3}{16} \text{ ft.};$$

$$\therefore \text{ Moment force applied to the large axle} = 400 \times \tfrac{1}{4} = 100;$$

$$\text{,, ,, ,, ,, small axle}$$

$$= 400 \times \frac{3}{16} = \frac{100 \times 3}{4} = \frac{300}{4} = 75;$$

$$\text{Moment force applied to the handle} = 2 \times P;$$

$$\therefore 2 \times P + 75 = 100;$$

$$\therefore P = \frac{100 - 75}{2} = \frac{25}{2} = 12{\cdot}5 \text{ lbs.}$$

Ex. 3.

Supposing the handle to make one revolution, we have Space moved over by W in ft. $= \tfrac{1}{2}$ of $\{\tfrac{1}{4} \times 3{\cdot}1416 - \tfrac{1}{8} \times 3{\cdot}1416\}$

$$= 3{\cdot}1416 \times \tfrac{1}{16};$$

$$\therefore \text{ Work of } W = 800 \times 3{\cdot}1416 \times \tfrac{1}{16}.$$

But from the solution to Ex. 1., we have

$$\text{Work of } P = 4 \times 3{\cdot}1416 \times P;$$

$$\therefore 4 \times 3{\cdot}1416 \times P = 800 \times 3{\cdot}1416 \times \tfrac{1}{16};$$

$$\therefore P = \frac{800}{4 \times 16} = 12{\cdot}5 \text{ lbs.}$$

Ex. 4.

First, on the principle of work. Here, proceeding as in Ex. 1., we have

$$\text{Space moved over by } P = 2 \times 2 \times 3{\cdot}1416 \text{ ft.};$$

$$\text{,, ,, ,, by } W = 3{\cdot}1416 \times \tfrac{3}{16} \text{ ft.};$$

$$\therefore \text{ Work of } P = 75 \times 4 \times 3{\cdot}1416;$$

$$\text{,, of } W = W \times 3{\cdot}1416 \times \tfrac{3}{16};$$

$$\therefore W \times 3{\cdot}1416 \times \tfrac{3}{16} = 75 \times 4 \times 3{\cdot}1416;$$

$$\therefore W \times \tfrac{3}{16} = 75 \times 4;$$

$$\therefore W = \frac{75 \times 4 \times 16}{3} = 25 \times 4 \times 16$$

$$= 1600 \text{ lbs.}$$

Second, on the principle of the equality of moments.

Moment force applied to the large axle $= \frac{1}{2}\,W \times \frac{1}{4} = \frac{1}{8}\,W$;

„ „ „ „ small axle $= \frac{1}{2}\,W \times \frac{1}{16} = \frac{1}{32}\,W$;

Moment force applied to the handle $= 75 \times 2 = 150$.

But as the sum of the last two moments is equal to the first, we have

$$\frac{1}{8}\,W = \frac{1}{32}\,W + 150;$$

$$\therefore\ \frac{1}{8}\,W - \frac{1}{32}\,W = 150,$$

$$W\left(\frac{1}{8} - \frac{1}{32}\right) = 150,$$

$$W \times \frac{3}{32} = 150;$$

$$\therefore\ W = \frac{150 \times 32}{3} = 50 \times 32 = 1600 \text{ lbs.}$$

Or thus, by the formula $W = \dfrac{4\,R \times P}{D - d}$.

Here, $R = 2$, $P = 75$ lbs., $D = \frac{1}{2}$, and $d = \frac{1}{8}$ ft.;

$$\therefore\ W = \frac{4 \times 2 \times 75}{\frac{1}{2} - \frac{1}{8}}$$

$$= \frac{4 \times 2 \times 75}{\frac{3}{8}} = \frac{4 \times 2 \times 75 \times 8}{3} = 1600 \text{ lbs.}$$

Art. 41.

Ex. 1.

Here the advantage gained by the pulleys is 8, that is, $8P = W$ or 16 lbs., therefore $P = \frac{1}{8}$ of 16 lbs. $= 2$ lbs.

Ex. 2.

Here the advantage gained is 16, that is, $16\,P = W$ or 16 lbs., therefore $P = \frac{1}{16}$ of 16 lbs. $= 1$ lb.

Ex. 3.

Here the tension of each cord by which the moveable pulley hangs $= \frac{1}{2}$ of $\{4 \text{ lbs.} + 180 \text{ lbs.}\} = \frac{1}{2}$ of 184 lbs. $= 92$ lbs., which must be the weight of P.

Ex. 4.

In this case, we have

Tension each cord by which the first moveable pulley is suspended $= \frac{1}{2}$ of $\{4 \text{ lbs.} + 180 \text{ lbs.}\} = 92$ lbs.

Tension each cord by which the second moveable pulley is suspended $= \frac{1}{2}$ of $\{4 \text{ lbs.} + 92 \text{ lbs.}\} = 48$ lbs., which must be the weight of P.

ART. 42.

Ex. 1.

Here the height of the plane being small as compared with its length, we have

Work in moving the body up the plane = wt. in lbs. × perpend. height $= 2 \times 2240 \times 2 = 8960$.

But we also have this work = pressure × length plane

$$= \text{pressure} \times 80.$$

$$\therefore \text{ Pressure} \times 80 = 8960;$$

$$\therefore \text{ Pressure} = \frac{8960}{80} = 112 \text{ lbs., or 1 cwt.}$$

Ex. 2.

In this case, we have

$$\text{Pressure} \times 80 = 2 \times 2240 \times 3;$$

$$\therefore \text{ Pressure} = \frac{2 \times 2240 \times 3}{80} = 168 \text{ lbs.}$$

$$= \frac{168}{112} \text{ cwts.} = 1\tfrac{1}{2} \text{ cwts.}$$

Ex. 3.

Let us suppose that the body is moved up the whole length of the plane by the pressure of P lbs.

Wt. body in lbs. $= 2 \times 2240 = 4480$ lbs.

Work due to friction $= \frac{1}{30}$ of 4480×80.

" " gravity $= 4480 \times 2$.

Now the sum of the works due to friction and gravity must be equal to the work of the pressure P, or $P \times 80$;

$$\therefore P \times 80 = \tfrac{1}{30} \text{ of } 4480 \times 80 + 4480 \times 2;$$

$$\therefore P = \tfrac{1}{30} \times 4480 + 112 = 149\tfrac{1}{3} + 112$$
$$= 261\tfrac{1}{3} \text{ lbs.}$$

Ex. 4.

In this case, we have

Work due to friction $= \frac{1}{20}$ of 4480×80

$$= 224 \times 80 = 17920;$$

$$\therefore P \times 80 = 17920 + 4480 \times 2$$
$$= 26880;$$

$$\therefore P = \frac{26880}{80} = 336 \text{ lbs., or } 3 \text{ cwt.}$$

ARTS. 44. AND 45.

Ex. 1.

Let the lever be turned round once; then

Space moved over by $P = 6 \times 3{\cdot}1416$,

" " " W = thickness thread

$$= \tfrac{1}{12} \text{ of } \tfrac{1}{2} = \tfrac{1}{24} \text{ ft.};$$

$$\therefore \text{Work of } P = 60 \times 6 \times 3{\cdot}1416;$$

Work of $W = W \times \frac{1}{24}$;

$$\therefore W \times \tfrac{1}{24} = 60 \times 6 \times 3{\cdot}1416;$$

$$\therefore W = 60 \times 6 \times 3{\cdot}1416 \times 24 = 27143 \text{ lbs.}$$

$$= \frac{27143}{2240} \text{ tons} = 12{\cdot}1 \text{ tons.}$$

Ex. 2.

Space moved over by $P = 4 \times 3{\cdot}1416$ ft.
$= 12{\cdot}5664$ ft.

Space moved over by $W = \frac{3}{4} \times \frac{1}{12}$ ft. $= \frac{1}{16}$ ft.;

$\therefore$ Work of $P = 60 \times 12{\cdot}5664 = 753{\cdot}984$;

Work of $W = W \times \frac{1}{16}$;

$$\therefore\ W \times \tfrac{1}{16} = 753{\cdot}984;$$

$$\therefore\ W = 753{\cdot}984 \times 16 = 12063{\cdot}744 \text{ lbs.}$$

$$= \frac{12063{\cdot}744}{2240} \text{ tons} = 5{\cdot}3 \text{ tons.}$$

Ex. 3.

In this case the expression for the work of P is the same as in Ex. 1.

Space moved over by $W = \frac{2}{3} \times \frac{1}{12} = \frac{1}{18}$ ft.

$$\therefore\ \text{Work of } W = W \times \tfrac{1}{18};$$

$$\therefore\ W \times \tfrac{1}{18} = 60 \times 6 \times 3{\cdot}1416;$$

$$\therefore\ W = 60 \times 6 \times 3{\cdot}1416 \times 18 = 20357{\cdot}568 \text{ lbs.}$$

$$= \frac{20357{\cdot}568}{2240} \text{ tons} = 9 \text{ tons.} +$$

Ex. 4.

Let the lever be turned round once; then
Space moved over by the press board

$$= \tfrac{1}{2} - \tfrac{3}{7} = \tfrac{1}{14} \text{ in.} = \tfrac{1}{168} \text{ ft.}$$

Work resistance, or work of $W = W \times \dfrac{1}{168}$.

Space moved over by $P = 6 \times 3{\cdot}1416$;

Work applied, or work of $P = 60 \times 6 \times 3{\cdot}1416$
$= 1130{\cdot}976$;

$$\therefore\ W \times \frac{1}{168} = 1130{\cdot}976;$$

$$\therefore\ W = 1130{\cdot}976 \times 168 = 190003{\cdot}968 \text{ lbs.}$$
$$= \frac{190003{\cdot}968}{2240} \text{ tons} = 84{\cdot}8 \text{ tons.}$$

Ex. 5.

Space moved over by the press board

$$= \tfrac{1}{2} - \tfrac{3}{8} = \tfrac{1}{8} \text{ in.} = \tfrac{1}{96} \text{ ft.};$$

$$\therefore\ \text{Work resistance} = W \times \frac{1}{96}.$$

Space moved over by $P = 6 \times 3{\cdot}1416$;

Work applied, or work $P = 20 \times 6 \times 3{\cdot}1416$

$$= 376{\cdot}992;$$

$$\therefore\ W \times \frac{1}{96} = 376{\cdot}992.$$

$$\therefore\ W = 376{\cdot}992 \times 96 = 36191{\cdot}232 \text{ lbs};$$
$$= \frac{36191{\cdot}232}{2240} \text{ tons} = 16{\cdot}1 \text{ tons.}$$

Ex. 6.

Here, $P = 60$ lbs., and $W = 190003$ lbs.

$$\therefore\ \text{Advantage} = \frac{W}{P} = \frac{190003}{60} = 3165+.$$

ART. 47.

Ex. 1.

Advantage gained by the lever $= \frac{30}{2} = 15$.

Therefore a pressure of 56 lbs. applied to the lever will produce a pressure of 15 times 56 lbs. or 840 lbs. upon the small piston.

Advantage gained by the pistons $= \frac{200}{1} = 200$.

Therefore a pressure of 840 lbs. applied to the small piston will produce a pressure of 200 times 840 lbs., or 168000 lbs., upon the large piston.

But 168000 lbs. $= \frac{168000}{2240}$ tons $= 75$ tons.

Ex. 2.

Here, as in the last example, the pressure exerted on the small piston is 840 lbs.; now we have

$$\text{Advantage gained by the pistons} = \frac{40}{1} = 40.$$

Therefore a pressure of 840 lbs. applied to the small piston will produce a pressure of 40 times 840 lbs., or 33600 lbs., upon the large piston.

But 33600 lbs. $= \frac{33600}{2240}$ tons $= 15$ tons.

Ex. 3.

Here, as in Ex. 1., the pressure exerted upon the small piston is 840 lbs.; now we have

$$\text{Advantage gained by the pistons} = \frac{400}{\frac{2}{3}}$$
$$= \frac{400 \times 3}{2} = 600;$$

$\therefore$ Pressure produced upon the large piston
$= 600$ times 840 lbs. $= 504000$ lbs. $= 225$ tons.

ARTS. 50. AND 52.

Ex. 1.

Here, velocity acquired in 1 sec. $= 32\frac{1}{6}$ ft.
$\therefore$ „ „ 5 sec. $= 5$ times $32\frac{1}{6}$ ft.
$= 160\frac{5}{6}$.

Or thus, by the formula, $v = t \times 32\frac{1}{6}$.
In this example, $t = 5$, and v is required;
$$\therefore \; v = 5 \times 32\tfrac{1}{6} = 160\tfrac{5}{6} \text{ ft.}$$

Ex. 2.

Velocity acquired in 1 sec. $=32\frac{1}{6}$ ft.
„ „ 2 sec. $=2$ times $32\frac{1}{6}$ ft.
$=64\frac{1}{3}$ ft.

Ex. 3.

In every second the velocity acquired, or gained, is $32\frac{1}{6}$ ft., therefore as many times as $32\frac{1}{6}$ ft. can be taken out of 193 ft. for so many seconds must the body have fallen;

$$\therefore \text{ No. seconds} = \frac{193}{32\frac{1}{6}} = 6 \text{ sec.}$$

Or thus, by the formula $v = t \times 32\frac{1}{6}$.
In this example, $v = 193$, and t is required;

$$\therefore\ t \times 32\frac{1}{6} = 193;$$

$$\therefore\ t = \frac{193}{32\frac{1}{6}} = 6 \text{ sec.}$$

Ex. 4.

$$\text{No. seconds} = \frac{386}{32\frac{1}{6}} = \frac{386 \times 6}{193} = 12.$$

Ex. 5.

$$\text{Space} = t^2 \times 16\tfrac{1}{12}$$
$$= 2^2 \times 16\tfrac{1}{12} = 64\tfrac{1}{3} \text{ ft.,}$$

which is the height of the tower.

Ex. 6.

$$\text{Space} = t^2 \times 16\tfrac{1}{12}$$
$$= 5^2 \times 16\tfrac{1}{12} = 25 \times 16\tfrac{1}{12}$$
$$= 402\tfrac{1}{12} \text{ ft.,}$$

which is the height required.

Ex. 7.

Here we shall first find the number of seconds which the body must have fallen to acquire the velocity of $96\frac{1}{2}$ ft.

$$\text{No. seconds in falling} = \frac{96\frac{1}{2}}{32\frac{1}{6}} = 3 \text{ sec.};$$

$$\therefore \text{ Space} = 3^2 \times 16\tfrac{1}{12} = 144\tfrac{3}{4} \text{ ft.};$$

which is the height to which the body will ascend.

Ex. 8.

In order to ascertain the height to which the body will rise in 2 seconds, we must find the space through which the body would fall during the last two seconds of its descent.

Space in 3 seconds $= 3^2 \times 16\frac{1}{12} = 144\frac{3}{4}$ ft.

„ „ 1 second $= 16\frac{1}{12}$;

$\therefore$ Space moved over during the last two seconds

$$= 144\tfrac{3}{4} - 16\tfrac{1}{12} = 128\tfrac{2}{3} \text{ ft.}$$

Ex. 9.

$$\text{No. seconds in falling} = \frac{193}{32\frac{1}{6}} = 6 \text{ sec.};$$

$$\therefore \text{ Space} = 6^2 \times 16\tfrac{1}{12} = 579 \text{ ft.},$$

which is the height from which the body must fall.

Ex. 10.

Accumulated work $= 8 \times 579 = 4632$.

Or thus by the general formula,

$$U = \frac{V^2 \times W}{2g}.$$

In this example, $V = 193$, $W = 8$;

$$\therefore \ U = \frac{193^2 \times 8}{2 \times 32\frac{1}{6}} = 4632,$$

which is the same result as before.

Ex. 11.

Here we shall first find the height from which the body must fall in order to acquire the given velocity.

No. seconds in falling $= \frac{386}{32\frac{1}{6}} = 12$ sec.;

$$\therefore \text{ Height} = 12^2 \times 16\tfrac{1}{12} = 2316 \text{ ft.};$$

$\therefore$ Work accumulated in the body = height in feet from which it must fall × wt. in lbs. $= 2316 \times 4 = 9264$.

Or thus by the general formula.

In this case, $V = 386$, and $W = 4$;

$$\therefore\ U = \frac{386^2 \times 4}{2 \times 32\frac{1}{6}} = 9264.$$

ARTS. 70. AND 71.

Ex. 1.

Construct the figure as directed in Art. 70. of the text; take $DC = 4$, and complete the parallelogram of pressures CQFD; then the units in DQ will be found to be 5·6, which are the units of cwts. pressure upon the pin.

Ex. 2.

In this case, take $DC = 3$, and construct the parallelogram CQFD; then the units in DQ will be found to be 4·2, which gives the pressure on the pin in units of cwts.

Ex. 3.

Here, $OA = 5$, $OH = 6$, and $DC = 4$; hence we find the units in $DQ = 4{\cdot}6$, which gives the pressure on the pin in units of cwts.

Ex. 4.

Here the wt. of the pier $= 5 \times 4 \times 1 \times 120$

$= 2400$ lbs.

Having constructed the section of the pier, &c., as directed in Art. **71.**, from any scale take $AC = 3$ units, and $AB = 2{\cdot}4$ units; complete the parallelogram ABDC, and produce the diagonal AD until it intersect the base OV in the point Q. Now as the point Q falls within the base of the pier, the structure will stand.

Ex. 5.

Having constructed the section of the pier, &c., as in the last example; join A and O; from any scale take $AB = 2{\cdot}4$ units, which in this case are units of thousands; complete the parallelogram of pressures; then the units in the direction of the pressure P will be found to be about 3·4, which gives the pressure P in units of thousands of lbs., that is, $P = 3400$ lbs.

Ex. 6.

Proceeding as in Ex. 4., it will be found that the point Q falls without the base; hence it follows that the pier will fall.

ARTS. 76. AND 78.

Ex. 1.

$$\text{Area surface upon which the water presses} = 10 \times 6 = 60 \text{ sq. ft.}$$

$$\therefore \text{ Pressure water} = 60 \times \tfrac{1}{2} \text{ of } 6 \times 62{\cdot}5 = 60 \times 3 \times 62{\cdot}5 = 11250 \text{ lbs.}$$

Ex. 2.

$$\text{Area surface upon which the water presses} = 10 \times 9 = 90 \text{ sq. ft.}$$

$$\therefore \text{ Pressure water} = 90 \times \tfrac{1}{2} \text{ of } 9 \times 62{\cdot}5 = 25312 \text{ lbs.}$$

Ex. 3.

Distance centre pressure from the bottom of the vessel

$=\frac{1}{3}$ of 6 ft. $=2$ ft.

Ex. 4.

Surface upon which the water presses $=9\times 1$

$=9$ sq. feet;

$\therefore$ Pressure of the water $=9\times\frac{1}{2}$ of $9\times 62{\cdot}5$

$=2531{\cdot}25$ lbs.

Distance centre of pressure from the bottom, or OI $=\frac{1}{3}$ of 9 ft. $=3$ ft. (See *fig.* p. 116. of the text.)

$\therefore$ Moment of the pressure of the water

$=$ Pressure water $\times$ OI $=2531{\cdot}25\times 3=7593{\cdot}75$.

Weight of the embankment $=9\times 4\times 1\times 120$

$=4320$ lbs.;

$\therefore$ Moment of the wt. of the embankment

$=$ wt. $\times$ OV $=4320\times\frac{1}{2}$ of $4=8640$.

Now since the moment of the weight of the embankment is greater than the moment of the pressure of the water, we conclude that the embankment will stand.

Ex. 5.

Here the moment of the pressure of the water is 7593·75, as in the last example.

Weight of the embankment $=9\times 3\times 1\times 120$

$=3240$ lbs.

$\therefore$ Moment of the wt. of the embankment

$=$ wt. $\times$ OV $=3240\times\frac{1}{2}$ of $3=4860$.

Here, as the latter moment is less than the former, we conclude that the embankment will fall.

ART. 82.

Ex. 1.

Here we have for the interior dimensions of the barge,

$$\begin{aligned}\text{Length} &= 10\text{ ft.}-2\times 2\text{ in.} = 10\text{ ft.}-4\text{ in.} = 9\tfrac{2}{3}\text{ ft.},\\ \text{Breadth} &= 4\text{ ft.}-2\times 2\text{ in.} = 4\text{ ft.}-4\text{ in.} = 3\tfrac{2}{3}\text{ ft.},\\ \text{Depth} &= 3\text{ ft.}-2\text{ in.} = 3\text{ ft.}-\tfrac{1}{6}\text{ ft.} = 2\tfrac{5}{6}\text{ ft.}\end{aligned}$$

$$\text{Content exterior solid} = 10\times 4\times 3 = 120\text{ c. ft.}$$

$$\text{,, interior ,,} = 9\tfrac{2}{3}\times 3\tfrac{2}{3}\times 2\tfrac{5}{6} = \frac{29\times 11\times 17}{3\times 3\times 6} = 100{\cdot}4;$$

$$\therefore\ \text{Timber in the barge} = 120-100{\cdot}4 = 19{\cdot}6\text{ c. ft.}$$

$$\text{Wt. of timber in the barge} = 19{\cdot}6\times 50 = 980\text{ lbs.}$$

In this case the displaced water is equal to the whole exterior volume of the barge.

$$\therefore\ \text{Wt. of the displaced water} = 120\times 62{\cdot}5 = 7500\text{ lbs.}$$

Hence we have,

$$\text{Load}+\text{wt. barge} = \text{wt. displaced water};$$

$$\therefore\ \text{Load}+980\text{ lbs.} = 7500\text{ lbs.};$$

$$\therefore\ \text{Load} = 7500-980 = 6520\text{ lbs.} = \frac{6520}{2240}\text{ tons} = 2{\cdot}9+\text{ tons.}$$

Ex. 2.

Here the weight of the barge is the same as in the last example.

$$\text{Load in lbs.} = 2\times 2240 = 4480\text{ lbs.};$$

$$\text{Wt. displaced water} = 10\times 4\times \text{depth}\times 62{\cdot}5\text{ lbs.};$$

$$\therefore\ 10\times 4\times \text{depth}\times 62{\cdot}5 = 980+4480;$$

$$\therefore\ \text{Depth} = \frac{980+4480}{10\times 4\times 62{\cdot}5} = 2{\cdot}18\text{ ft.}$$

MISCELLANEOUS EXAMPLES.

Ex. 1.

Work engine per hour $= 17894000$;

$$\therefore \text{ Work engine per min.} = \frac{17894000}{60};$$

$$\therefore \text{ H. P.} = \frac{17894000}{60 \times 33000} = 9{\cdot}03.$$

Ex. 2.

Effective work engine per min. $= 7 \times 33000 \times \frac{2}{3}$;

$\therefore$ „ „ „ hour $= 7 \times 33000 \times \frac{2}{3} \times 60$

$= 7 \times 11000 \times 2 \times 60 = 9240000.$

Work in raising 1 c. ft. of water $= 62{\cdot}5 \times 100 = 6250$;

$$\therefore \text{ No. c. ft. water raised} = \frac{9240000}{6250} = 1478{\cdot}4.$$

Ex. 3.

Effective work engine per min. $= 80 \times 33000$;

Volume water raised each stroke $= 2 \times 5 = 10$ c. ft.;

Wt. water raised each stroke $= 10 \times 62{\cdot}5 = 625$ lbs.;

Height to which the water is raised $= 60 \times 6 = 360$ ft.;

$\therefore$ Work in each stroke $= 625 \times 360$;

$$\therefore \text{ No. strokes per min.} = \frac{\text{work engine per min.}}{\text{work in each stroke}}$$

$$= \frac{80 \times 33000}{625 \times 360} = 11{\cdot}7 + .$$

Ex. 4.

Content of the material $= 30 \times 40 \times 6$ c. ft.;

$\therefore$ Wt. of the material $= 30 \times 40 \times 6 \times 100 = 720000$ lbs.;

$$\therefore \text{ Work in raising the material} = 720000 \times \frac{40 \times 6}{2}$$
$$= 86400000.$$

Ex. 5.

Content of the material $= 30 \times 5 \times 6$ c. ft.;

$\therefore$ Wt. of the material $= 30 \times 5 \times 6 \times 100 = 90000$ lbs.;

$$\therefore \text{ Work in raising the material} = 90000 \times \frac{5 \times 6}{2}$$
$$= 90000 \times 15 = 1350000.$$

Ex. 6.

No. c. ft. in the wall $= 40 \times 20 \times 3 = 2400$;

$\therefore$ Wt. of the wall $= 2400 \times 120 = 288000$ lbs.;

$$\therefore \text{ Work in raising the material} = 288000 \times \frac{20}{2}$$
$$= 2880000.$$

Ex. 7.

No. c. ft. water in the cistern $= 12 \times 6 \times 10 = 720$;

$\therefore$ Wt. water in the cistern $= 62{\cdot}5 \times 720 = 45000$ lbs.;

Height to which the centre of gravity of the water in the cistern is raised $= 30 + \frac{10}{2} = 35$ ft.;

$$\therefore \text{ Work in raising the water} = 45000 \times 35$$
$$= 1575000.$$

Ex. 8.

No. c. ft. water in the cistern $= 12 \times 6 \times 12 = 864$;

$$\therefore \text{ Wt. water in the cistern} = 62{\cdot}5 \times 864$$
$$= 54000 \text{ lbs.}$$

Height to which the centre of gravity of the water in the cistern is raised $=30+\frac{12}{2}=36$ ft.;

$$\therefore \text{ Work in raising the water} = 54000 \times 36 = 1944000.$$

Ex. 9.

Work per hour $=1575000 \times 3$;

$$\therefore \text{ Work per min.} = \frac{1575000 \times 3}{60};$$

$$\therefore \text{ H. P.} = \frac{1575000 \times 3}{60 \times 33000} = 2{\cdot}38+.$$

Ex. 10.

Space moved over by the horse in each revolution $=20 \times 3{\cdot}1416 = 62{\cdot}832$ ft.;

$\therefore$ Work of the horse in each revolution $=160 \times 62{\cdot}832$.

But the work of the horse per min. $=22000$;

$$\therefore \text{ No. revolutions of the horse per min.} = \frac{22000}{160 \times 62{\cdot}832} = 2{\cdot}18.$$

Ex. 11.

Let $x=$ the distance of the centre of gravity from W; then $15-x=$ the distance from P; hence we have, by the equality of moments,

$$\text{Moment W} = 8 \times x;$$

$$\text{Moment P} = 4 \times (15-x);$$

$$\therefore 8 \times x = 4 \times (15-x);$$

$$\therefore 8 \times x = 60-4x;$$

$$\therefore 12x = 60, \text{ and } x = \frac{60}{12} = 5 \text{ in.}$$

Ex. 12.

Here we have,

$$\text{Moment } W = 8 \times x\,;$$
$$\text{Moment } P = 2 \times (15 - x)\,;$$
$$\therefore\ 8 \times x = 2 \times (15 - x)\,;$$
$$\therefore\ 10 \times x = 30, \text{ and } x = 3 \text{ in.}$$

THE END.

London:
Spottiswoodes and Shaw,
New-street-Square.

ESTABLISHED SCHOOL BOOKS.

I.

Dr. L. SCHMITZ's SCHOOL ABRIDGMENT of BISHOP THIRLWALL's HISTORY of GREECE. *Second Edition.* 12mo. 7s. 6d.

II.

BARKER's SCHOOL ABRIDGMENT of LEMPRIERE's CLASSICAL DICTIONARY; revised and corrected by Dr. J. CAUVIN. 8vo. 12s.

III.

BLAIR's CHRONOLOGICAL and HISTORICAL TABLES, corrected and extended under the revision of Sir H. ELLIS. *New Edition.* Imperial 8vo. 31s. 6d.

IV.

BRADLEY's PHÆDRUS, with English Notes, &c. *New Edition*, corrected and improved, by the Rev. JOHN T. WHITE, M.A. 12mo. 2s. 6d.

V.

BRADLEY's OVID'S METAMORPHOSES, with English Notes, &c. *New Edition*, corrected and improved, by the Rev. J. T. WHITE, M.A. 12mo. 4s. 6d.

VI.

BRADLEY's EUTROPIUS, with English Notes, &c. *New Edition*, corrected and improved, by the Rev. J. T. WHITE, M.A. 12mo. 2s. 6d.

VII.

BRADLEY's CORNELIUS NEPOS, with English Notes, &c. *New Edition*, corrected and improved, by the Rev. J. T. WHITE, M.A. 12mo. 3s. 6d.

VIII.

The Rev. J. T. WHITE's SCHOOL EDITION of TACITUS's GERMANY and AGRICOLA, with English Notes, &c. 12mo. 4s. 6d.

IX.

The Rev. J. T. WHITE's SCHOOL EDITION of XENOPHON's ANABASIS, with English Notes, &c. 12mo. 7s. 6d.

X.

The Rev. Dr. KENNEDY's ELEMENTARY GRAMMAR of the LATIN LANGUAGE. *New Edition.* 12mo. 3s. 6d.

London: LONGMAN, BROWN, GREEN, and LONGMANS.